Mathematics: Term 2 Coursework 1

To be handed in to CM520 by 4:00pm on Monday 21 March 2005

1. ρ is a relation defined on the set M of 2×2 matrices. A is a given 2×2 matrix. For any two matrices X, $\mathsf{Y} \in M$, $\mathsf{X}\ \rho\ \mathsf{Y}$ if there is some real number k such that $\mathsf{X} - \mathsf{Y} = k\mathsf{A}$. Prove whether or not ρ is an equivalence relation.

2. M is the matrix given by

$$\mathsf{M} = \begin{pmatrix} 3 & 1 & -3 \\ 1 & 2a & 1 \\ 0 & 2 & a \end{pmatrix}.$$

   (i) Find the values of a for which M is singular.

   (ii) Find the inverse matrix of M when $a = 1$.

3. In lecture we derived the result

$$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$$

   using complex variable techniques. Derive using the same method an equivalent relation for $\sin^4\theta$.

4. Use the method of differences to find the sum of the first n terms of the following series

$$S_n = 2 + 3x + 4x^2 + 5x^3 + \cdots$$

5. Solve the difference equations:

   (i) $u_{n+1} = 3u_n + 4, \qquad n = 0, 1, 2, ...;$

   (ii) $u_{n+2} = 2u_{n+1} + 8u_n$, $n = 0, 1, 2, ...$ where $u_0 = -1$ and $u_1 = 8$.

6. There may be a *few* bonus marks for anyone who solves the "Cost of Beer" problem.