MATHEMATICS: TERM 2 COURSEWORK 1

TO BE HANDED IN TO CM520
BY 4:00PM ON MONDAY 13 MARCH 2006

1. ρ is a relation defined on the set M of 2×2 matrices. A is a given 2×2 matrix. For any two matrices X, $\mathsf{Y} \in M$, $\mathsf{X} \, \rho \, \mathsf{Y}$ if there is some real number k such that $\mathsf{X} - \mathsf{Y} = k\mathsf{A}$. Prove whether or not ρ is an equivalence relation.

2. M is the matrix given by

$$\mathsf{M} = \begin{pmatrix} a & 2 & 0 \\ 1 & 2a & 1 \\ -3 & 1 & 3 \end{pmatrix}.$$

   (i) Find the values of a for which M is singular.

   (ii) Find the inverse matrix of M when $a = 1$.

3. In lectures we derived the result

$$\cos^4 \theta = \frac{1}{8} \cos 4\theta + \frac{1}{2} \cos 2\theta + \frac{3}{8}$$

   using complex variable techniques. Derive using the same method an equivalent relation for $\sin^5 \theta$.

4. Use the method of differences to find the sum of the first n terms of the following series

$$S_n = 5 + 7x + 9x^2 + 11x^3 + \cdots$$

5. Solve the difference equations:

   (i) $u_{n+1} = 2u_n + 2 - n$, $n = 0, 1, 2, \ldots$where $u_0 = 1$

   (ii) $u_{n+2} = 3u_{n+1} + 4u_n$, $n = 0, 1, 2, \ldots$ where $u_0 = 0$ and $u_1 = 1$.