Mathematics: Term 2 Coursework 2

To be handed in to CM520
by 4:00pm on Monday 25 April 2005

1. Using partial differentiation find $\frac{dy}{dx}$ where

    (i) $x^2 + y^2 + \tanh xy = C$,

    (ii) $(x^2 + y^2)e^{xy} = C$.

2. Find all four stationary points (i.e., points where $f_x = f_y = 0$) of

$$f(x, y) = x^2 y + xy^2 - x^2 - y^2 - 3xy + 2x + 2y,$$

    and find their natures (i.e., are they maxima, minima or saddle points?)

3. Solve the following first order ordinary differential equations:

    (i) $(3y^2 + xe^{xy})\dfrac{dy}{dx} + 3x^2 + ye^{xy} = 0$.

    (ii) $\dfrac{dy}{dx} - x^3 y = 0$.

4. Solve the following first order ordinary differential equations:

    (i) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} = e^{-3x} - 1, \quad y(0) = y'(0) = 0$,

    (ii) $x^2 \dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = 4x^3$.

5. Evaluate the following integral using a reduction method:

$$I_n = \int_0^1 x(\ln x)^n \, dx.$$

6. Find the length of the curve $y^2 = x^3$ between $x = y = 0$ and $x = y = 1$.

Reminder: You will not be getting tables of integrals and derivatives in your examinations this year. Copies of the sheets used previously for the Mathematics I examination are available on the web page below. You are expected to know the content of these two pages.

Copies of all handouts can be found at
http://www.staff.city.ac.uk/o.s.kerr/ActSciMaths/