Mathematics: Term 2 Questions 1a
More Relations

1. *From 2005 Summer examination*

   (a) A relation $\sim$ is defined on the set $A = \{a, b, c, d\}$. Give the definitions of symmetric, reflexive and transitive relations.

   Say whether the following relations are are symmetric, reflexive or transitive, giving reasons:

   (i) $\sim = \{(a, a), (a, d), (b, b), (c, c), (d, a), (d, d)\}$,

   (ii) $\sim = \{(a, a), (a, c), (b, b), (c, a), (c, c)\}$,

   (iii) $\sim = \{(a, a), (a, b), (b, a), (b, c), (c, b), (c, c), (d, d)\}$.

   (b) For any set A, the power set $P(A)$ is the set of all subsets of A.

   (i) Write down all the elements of $P(A)$ when $A = \{0, 1, 2\}$.

   (ii) Draw a Hasse diagram for the partially ordered set $(P(A), \subseteq)$

   (iii) Give the lower bounds for the subset of $P(A)$ given by $\{\{0, 1\}, \{0, 2\}\}$. What is the greatest lower bound?

2. ρ is a relation defined on the set M of 2×2 matrices. A is a given 2×2 matrix. For any two matrices X, $\mathsf{Y} \in M$, $\mathsf{X}\ \rho\ \mathsf{Y}$ if there is some real number k such that $\mathsf{X} - \mathsf{Y} = k\mathsf{A}$. Prove whether or not ρ is an equivalence relation.

Solutions

1. (a) The definitions are:
   - **Reflexive**: $x \sim x$ for all x.
   - **Symmetric**: If $x \sim y$ then $y \sim x$.
   - **Transitive**: If $x \sim y$ and $y \sim z$ then $x \sim z$.

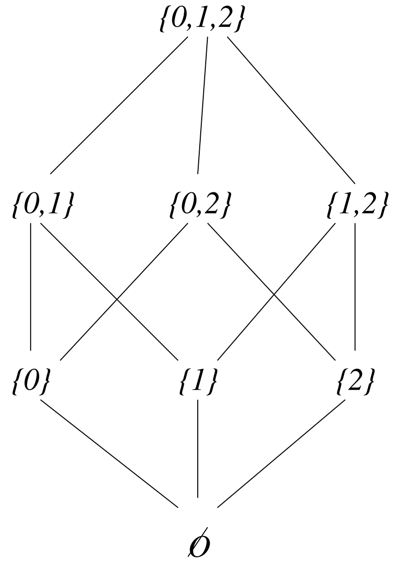
   (i) Reflexive, symmetric and transitive

   (ii) Not reflexive (no (d, d)), but is symmetric and transitive

   (iii) Not reflexive (no (b, b)), is symmetric, but not transitive (have (a, b) and (b, c), but no (a, c))

   (b) (i) $\emptyset$, $\{0\}$, $\{1\}$, $\{2\}$, $\{0, 1\}$, $\{1, 2\}$, $\{0, 2\}$, $\{0, 1, 2\}$

   (ii) The Hasse diagram is



   (iii) Lower bounds are $\emptyset$ and $\{0\}$. Greatest lower bound is $\{0\}$.

2. The relation is an equivalence relation as it is:

   (i) Reflexive: For all X, $\mathsf{X} - \mathsf{X} = 0\mathsf{A}$

   (ii) Symmetric: If $\mathsf{X}\ \rho\ \mathsf{Y}$ then there exists k such that $\mathsf{X} - \mathsf{Y} = k\mathsf{A}$. Then $\mathsf{Y} - \mathsf{X} = (-k)\mathsf{A}$ so $\mathsf{Y}\ \rho\ \mathsf{X}$.

   (iii) Transitive: If $\mathsf{X}\ \rho\ \mathsf{Y}$ and $\mathsf{Y}\ \rho\ \mathsf{Z}$ then there exist k, k' such that $\mathsf{X} - \mathsf{Y} = k\mathsf{A}$ and $\mathsf{Y} - \mathsf{Z} = k'\mathsf{A}$. Then $\mathsf{X} - \mathsf{Z} = (k + k')\mathsf{A}$ so $\mathsf{X}\ \rho\ \mathsf{Z}$.