Mathematics: Term 2 Questions 3
More Matrices

1. Verify:

   (a) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda & 1 \end{vmatrix} = 1$ (b) $\begin{vmatrix} 1 & 0 & \lambda \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

   (c) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -1$ (d) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \alpha$

   (e) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}$ (f) $\begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}$

2. Write down some 3×3 matrix and verify that pre-multiplying by the first 4 matrices in the previos question (a) adds $\lambda \times$ row 2 to row 3, (b) $\lambda \times$ row 3 to row 1, (c) swaps rows 2 and 3 and (d) multiplies row 2 by α.

3. Write the following systems of equations in matrix form, then write down the corresponding augmented matrix and solve using the Gaussian elimination/row reduction method:

   (a) $\begin{aligned} x + y &= 4 \\ 3x + 2y &= 9 \end{aligned}$ (b) $\begin{aligned} 4x - y &= 5 \\ 3x + y &= 9 \end{aligned}$

   (c) $\begin{aligned} x - 2y + z &= 6 \\ 2x - y + 4z &= 15 \\ x + 5y + z &= -1 \end{aligned}$ (d) $\begin{aligned} 2x - 2y + z &= 6 \\ x - y + 3z &= 8 \\ 4x + y + z &= -5 \end{aligned}$

4. Use row reduction techniques to find the inverses of the following matrices:

   (a) $\begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 0 \\ 1 & -2 & 0 \end{pmatrix}$

5. Use Cramers theorem to solve (a) and (c) in question 3 (and (b) and (d) if you are feeling keen).

6. Solve the system of equations in the “Cost of Beer” handout, verifying that the solutions given are (a) unique and (b) correct. Note: you can’t just insert the values given as this will not show uniqueness.

Solutions

1. —

2. —

3. (a) $x = 1, \quad y = 3$, (b) $x = 2, \quad y = 3$,
   (c) $x = 1, \quad y = -1, \quad z = 3$ (d) $x = -1, \quad y = -3, \quad z = 2$.

4.

(a) $\begin{pmatrix} 1 & 0 \\ 3/2 & -1/2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 1/2 & -1/2 \\ 1/4 & -3/8 & -1/8 \end{pmatrix}$

5. —

6. —