Mathematics: Term 2 Questions 6
Difference Equations

1. Find the general solution to the following first order difference equations

   (a) $A_{n+1} + 3A_n = 0$ (b) $A_{n+1} - 2A_n = 3^n$

   (c) $A_{n+1} - 2A_n = n + 1$ (d) $A_{n+1} - A_n = n + 1$

2. Find the solutions for question 1 if in each case $A_1 = 1$.

3. Find the general solution to the following second order difference equations

   (a) $A_{n+2} - 4A_{n+1} + 3A_n = 0$ (b) $A_{n+2} + 5A_{n+1} - 6A_n = 3^n$

   (c) $A_{n+2} + 2A_{n+1} - A_n = 0$ (d) $A_{n+2} + 2A_{n+1} + 5A_n = 2$

   (e) $A_{n+2} + 2A_{n+1} + 3A_n = 0$ (f) $A_{n+2} - 2A_{n+1} + A_n = n + 1$

4. Find the solutions to question 3 if $A_1 = A_2 = 1$.

5. Count the different sets of spirals in this picture of a sunflower:

Solutions

1. (a) $A_n = C(-3)^n$ (b) $A_n = 3^n + C2^n$
   (c) $A_n = -n - 2 + C2^n$ (d) $A_n = C + n/2 + n^2/2$

2. (a) $A_n = (-3)^{n-1}$ (b) $A_n = 3^n - 2^n$
   (c) $A_n = -n - 2 + 2^n$ (d) $A_n = n/2 + n^2/2$

3. (a) $A_n = B + C3^n$ (b) $A_n = 3^n/18 + B + C(-6)^n$
   (c) $A_n = B(-1+\sqrt{2})^n + C(-1-\sqrt{2})^n$ (d) $A_n = 1/4 + B(-1+2i)^n + C(-1-2i)^n$
   (e) $A_n = B(-1+i\sqrt{2})^n + C(-1-i\sqrt{2})^n$ (f) $A_n = n^3/6 + B + Cn$

4. (a) $A_n = 1$
   (b) $A_n = 3^n/18 + 11/14 + (-6)^n/126$
   (c) $A_n = \dfrac{3+\sqrt{2}}{2}(-1+\sqrt{2})^n + \dfrac{3-\sqrt{2}}{2}(-1-\sqrt{2})^n$
   (d) $A_n = 1/4 + \dfrac{-9-3i}{40}(-1+2i)^n + \dfrac{-9+3i}{40}(-1-2i)^n$
   (e) $A_n = -\dfrac{1}{2}(-1+i\sqrt{2})^n - \dfrac{1}{2}(-1-i\sqrt{2})^n$
   (f) $A_n = n^3/6 + 2 - 7n/6$

5. The most obvious two sets have 21 and 34 spirals — Fibonacci numbers.