Mathematics: Term 2 Questions 8
Differential Equations (First Order)

1. Which of the following are exact differential equations (i.e., of the form $\frac{dF}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$)

   (a) $2x + y^2 + 2xy\dfrac{dy}{dx} = 0$

   (b) $x^3 + y^3 + (3x^2 + 3xy^2)\dfrac{dy}{dx} = 0$

   (c) $4x + 2xy^2 + (2x^2y + 6y)\dfrac{dy}{dx} = 0$

   (d) $2xye^y + x^2(y+1)e^y\dfrac{dy}{dx} = 0$

2. For the answers to question 1 that are exact differential equations, integrate to find the solution (i.e., find $F = C$)

3. Use integrating factors to find the solutions to

   (a) $\dfrac{dy}{dx} + x^3y = 0$

   (b) $\dfrac{dy}{dx} + y\cot x = \cos^2 x$

4. Use separation of variables to find solutions to

   (a) $\dfrac{dy}{dx} + x^3y = 0$

   (b) $\dfrac{dy}{dx} + (2x+1)y^2 = 0$

   (c) $\dfrac{dy}{dx} + 2(y-1) = 0$

   (d) $\dfrac{dy}{dx} - y\tanh x = 0$

Solutions

1. (i) yes, (ii) no, (iii) yes, (iv) yes.

2. (a) $x^2 + xy^2 = C$ (b) —
   (c) $2x^2 + x^2y^2 + 3y^2 = C$ (d) $x^2ye^y = C$

3. (a) $R(x) = e^{x^4/4}, \quad y = Ce^{-x^4/4}$ (b) $R(x) = \sin x, \quad y = -\dfrac{\cos^3 x}{3\sin x}x + \dfrac{C}{\sin x}$

4. (a) $y = Ce^{-x^4/4}$ (b) $y = \dfrac{1}{x^2 + x + C}$
   (c) $y = Ce^{-2x} + 1$ (d) $y = C\cosh x$