Mathematics: Term 2 Questions 9
Differential Equations (Second Order)

1. Find the general solutions to the following differential equations:

   (i) $\dfrac{d^2y}{dx^2} - y = e^{3x}$

   (ii) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 2e^{x} - 5e^{2x}$

   (iii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 10\sin 2x$

   (iv) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = \cos 3x + \sin 3x$

   (v) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{-2x}$

   (vi) $\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 25y = 48\cos x - 16\sin x$

2. Find the solution to the initial value problem

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 36x - 10e^{2x}, \quad y(0) = -\frac{1}{2}, \quad y'(0) = -15.$$

3. Find the solution to the boundary value problem

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \left(x^2 + 2x - 1\right)e^{2x}, \quad y(0) = 0, \quad y(1) = 0.$$

4. Find the general solutions to the following differential equations:

   (i) $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = 4x^3$

   (ii) $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + y = 0$

Solutions

1. (i) $y = Ae^{x} + Be^{-x} + \dfrac{e^{3x}}{8}$

   (ii) $y = Ae^{x} + Be^{2x} - 2xe^{x} - 5xe^{2x}$

   (iii) $y = (A\cos x + B\sin x)e^{-x} - 2\cos 2x - \sin 2x$

   (iv) $y = A + Be^{x} - \dfrac{\cos 3x}{15} - \dfrac{2\sin 3x}{15}$

   (v) $y = Ae^{-x} + Be^{-2x} - xe^{-2x}$

   (vi) $y = (A\cos 3x + B\sin 3x)e^{-4x} + 2\cos x$

2. $y = -5e^{-2x} + 10e^{-3x} + 6x - 5 - \dfrac{e^{2x}}{2}$

3. $y = (x-1)e^{x} + (x-1)^{2}e^{2x}$

4. (i) $y = Ax + Bx^{2} + 2x^{3}$

   (ii) $y = \dfrac{A + B\ln x}{x}$