Computational Mathematics/Information Technology

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2009–10

Financial Functions in Excel

This lecture starts to develop the background for the financial functions in Excel that deal with, for example, loan repayments and other compound interest problems. As an example, if I borrow £5000 over three years at a fixed rate of interest how much do I pay back per month.

Geometric Progressions

Definition: A geometric progression or series of n values is an ordered set of the form:

$$a, \quad ar, \quad ar^2, \quad ar^3, \quad \ldots \quad , ar^{n-1}$$

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For example:

$$5, \quad \frac{5}{2}, \quad \frac{5}{4}, \quad \frac{5}{8}, \quad \ldots, \quad 5\left(\frac{1}{2}\right)^{n-1} \qquad a = 5 \quad r = \frac{1}{2}$$

$$3, \quad -\frac{3}{2}, \quad \frac{3}{4}, \quad -\frac{3}{8}, \quad \ldots, \quad \left(-\frac{1}{2}\right)^{n-1} \qquad a = 3 \quad r = -\frac{1}{2}$$

Usually we are interested in the sum of a geometric progression, a formula for which can be obtained as follows:

Let

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

then

$$S_n - rS_n = \left(a + ar + ar^2 + \cdots + ar^{n-1}\right) - \left(ar + ar^2 + ar^3 + \cdots + ar^n\right)$$

$$S_n - rS_n = a - ar^n$$

Thus we have

$$S_n(1-r) = a(1-r^n) \quad \Rightarrow \quad S_n = a\left(\frac{1-r^n}{1-r}\right)$$

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Thus we have

$$S_n(1-r) = a(1-r^n) \quad \Rightarrow \quad S_n = a\left(\frac{1-r^n}{1-r}\right) \quad r \neq 1$$

Note:

- If $r = 1$ the formula is not valid. However in this case
$$S_n = a + a + \cdots + a$$
where we have a appearing n times, and thus $S_n = na$.
- In interest problems $|r|$ is often less than 1 and quite often n is large; we thus note the following result:

If $|r| < 1$ then:

$$\text{as} \quad n \to \infty \quad r^n \to 0 \qquad \text{Thus} \quad S_n = a\left(\frac{1 - r^n}{1 - r}\right) \to \frac{a}{1 - r}$$

Indeed $\dfrac{a}{1 - r}$ is quite a common approximation for S_n when n is large and r is small.

Basic Compound Interest

The basic compound interest problem is:

If £A is invested for n periods at an interest rate of r per period, how much is the investment worth at the end of the n periods?

For example if I invest £1000 for 12 months compounded monthly at a rate of 1% at the end of each month how much do I have at the end of the year? In this example $A = 1000$, $r = 0.01$ and $n = 12$.

Note that r is expressed as a fraction in the mathematics even though it is expressed as a percentage in the question.

period	0	1	2	n
Amount	A	$A + rA$	$A(1 + r) + rA(1 + r)$	$A(1 + r)^n$
		$A(1 + r)$	$A(1 + r)^2$	

We have an initial investment of A over n periods, with the interest is added at the end of each period.

- At the end of the first month you have your initial investment, A, plus interest on the investment, rA, giving a total of $A(1 + r)$.

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- At the end of the second interval we have $A(1 + r)$ plus the interest on this amount, $rA(1 + r)$. giving a total $A(1 + r)^2$.

period	0	1	2	...	n
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- At the end of the second interval we have $A(1 + r)$ plus the interest on this amount, $rA(1 + r)$. giving a total $A(1 + r)^2$.
- At the end of the n^{th} period we have a total amount of $A(1 + r)^n$.

In our simple example of £1000 being invested at 1% per month for a year the amount at the end of the 12 months

$$S_{12} = 1000(1 + 0.01)^{12} = £1126.83.$$

Note: The final amount is £6.83 more than if the compounding had been carried out only once at the end of the year at a rate of 12%.

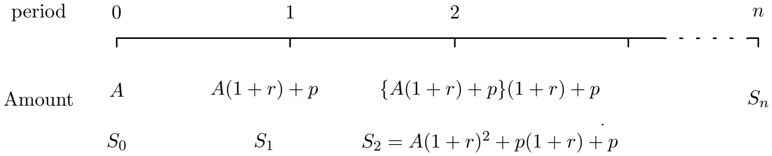
In all our problems it will be clear what the rate is and when we are applying it. We will not discuss the idea of effective interest rates or indeed the various ways in which banks express the interest rates in their advertising.

In summary:

An amount A invested at a rate r for n periods amounts to $A(1+r)^n$.

Basic Investment Problem

If an amount A is invested initially at a rate r for n periods and in addition we invest an amount p at the end of each period what will the final amount be?



- At the end of the first period the initial investment has grown to $A(1+r)$ Adding to this the regular investment p gives a total of

$$S_1 = A(1+r) + p$$

period	0	1	2	$\cdots$	n
Amount	A	$A(1+r)+p$	$\{A(1+r)+p\}(1+r)+p$		S_n
	S_0	S_1	$S_2 = A(1+r)^2 + p(1+r) + p$		

- At the end of the first period the initial investment has grown to $A(1+r)$ Adding to this the regular investment p gives a total of

$$S_1 = A(1+r) + p$$

- At the end of the second period this will grow to its original value multiplied by $(1+r)$, plus our next regular payment p, so

$$S_2 = \{A(1+r)+p\}(1+r) + p = A(1+r)^2 + p(1+r) + p$$

Carrying out this process once more gives:

$$S_3 = \left\{A(1+r)^2 + p(1+r) + p\right\}(1+r) + p$$

$$= A(1+r)^3 + p(1+r)^2 + p(1+r) + p$$

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$$= A(1+r)^3 + p(1+r)^2 + p(1+r) + p$$

From this we can see the n^{th} case will be

$$S_n = A(1+r)^n + p(1+r)^{n-1} + p(1+r)^{n-2} + \cdots + p$$

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We sum the geometric progression part (noting the common ratio is $(1+r)$):

$$S_n = A(1+r)^n + p\left\{\frac{1-(1+r)^n}{1-(1+r)}\right\}$$

$$= A(1+r)^n + p\frac{(1+r)^n - 1}{r}$$

In summary:

An initial investment A and regular investment p at the end of each period, compounded at the rate r per period, becomes after n periods:

$$S_n = A(1+r)^n + p\left\{\frac{(1+r)^n - 1}{r}\right\}$$

Example: I initially invest £1000 in a saving scheme and then at the end of each month I invest an extra £50. If the interest rate is 0.5% per month and I continue this process for two year, how much will my saving be worth?

Substituting directly into our equation with $A = 1000$, $p = 50$, $r = 0.005$ and $n = 24$ we obtain the final amount:

$$\text{final amount} = 1000(1+0.005)^{24} + 50\left\{\frac{(1+0.005)^{24}-1}{0.005}\right\}$$
$$= £2398.76$$

Basic Financial Worksheet Functions in Excel

Although the mathematical ideas behind such problems are quite straightforward some care needs to be employed when trying to implement the functions provided by Excel. Thus a thorough understanding of the mathematical formulas being used by Excel is essential in order to obtain and interpret correctly the answers given.

In the previous example it is clear how we should enter the parameters A, p, r, and n, however to use the worksheet functions in Excel we have to adopt a sign convention:

Excel's sign convention: Money that flows away from you is given a negative sign and money that flows towards you is given a positive sign.

- If you have to put your hand in your pocket and pay money out then it carries a minus sign.
- If money is put into your pocket then it carries a positive sign.

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We make an initial investment, thus in the Excel functions we would enter -1000 and not $+1000$.

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We make a monthly payment thus we would enter -50 into the Excel functions and not $+50$.

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We make an initial investment, thus in the Excel functions we would enter -1000 and not $+1000$.

We make a monthly payment thus we would enter -50 into the Excel functions and not $+50$.

The final amount is paid back to you thus Excel will yield a positive answer.

Denoting the final or future value by FV, monthly payments by PMT and the initial or present value of our investment by PV the sign convention would require:

- $A = -\text{PV}$ since the initial amount is paid out to the Bank
- $p = -\text{PMT}$ since the payments are paid out to the Bank
- $S_n = \text{FV}$ since the future value is paid back to you.

Recall

$$S_n = A(1+r)^n + p\left\{\frac{(1+r)^n - 1}{r}\right\}$$

Excel is using the formula:

$$\mathrm{FV} = -\mathrm{PV}(1+r)^n - \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

Excel's Future Value Function

=**FV**(r, n, PMT, PV, type)

The parameters are as in

$$\mathrm{FV} = -\mathrm{PV}(1+r)^n - \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

with the additional parameter "type". If type is set equal to zero then this is the formula used.

If type is set equal to 1 then it is assumed that the payments are made at the start of each period.

Previous Example: I initially invest £1000 in a saving scheme and then at the end of each month I invest an extra £50. If the interest rate is 0.5% per month and I continue this process for two year, how much will my saving be worth?

Using Excel we would enter into the worksheet:

$$= \mathbf{FV}(0.005, 24, -50, -1000, 0)$$

or

$$= \mathbf{FV}(0.5\%, 24, -50, -1000, 0)$$

This gives £2398.76 as before.

Excel's Present Value Function

=**PV**(r, n, PMT, FV, type)

This is

$$\mathrm{FV} = -\mathrm{PV}(1+r)^n - \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

rearranged to make PV the subject of the formula giving

$$\mathrm{PV} = -\left\{\mathrm{FV} + \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}\right\}(1+r)^{-n}$$

Example: If I wish to accumulate £5000 in four years time by depositing £75 per month in a fixed rate account with interest rate of 0.4% per month, what initial investment must I also make?

Here the accumulated amount of £5000 is the FV which is paid to us, thus it is entered as a positive quantity.

We are making payments of £75 thus we enter PMT as -75.

Four years is 48 periods thus $n = 48$.

The interest rate is entered as 0.004 or 0.4%.

We enter:

$$= \textbf{PV}(0.004,\ 48,\ -75,\ 5000,\ 0)$$

This gives -858.55.

The minus sign indicating that we must pay out, that is to say deposit £858.55, at the start of the investment in order to reach £5000 by the end.

Excel's Payment Function

$$=\textbf{PMT}(\text{r, n, PV, FV, type})$$

This is

$$\text{FV} = -\text{PV}(1+r)^n - \text{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

rearranged to make PMT the subject of the formula:

$$\text{PMT} = -\left\{\text{FV} + \text{PV}(1+r)^n\right\} \times \left\{\frac{r}{(1+r)^n - 1}\right\}$$

Example: How much will the monthly repayments be if I borrow £100,000 over 20 years with an effective monthly interest rate of 0.5%?

The final value, FV, is zero, since we must pay back all the loan and interest by the end of the period.

The loan is paid to us at the start, so the present value, PV is $+100,000$.

The number of payments is 240 and we assume that we pay at the end of each month thus type=0.

We enter:

$$= \textbf{PMT}(0.5\%,\ 240,\ 100000,\ 0,\ 0)$$

This gives -716.43.

As expected the result is negative since we are paying out money each month.

The last two Excel functions based explicitly around the equation

$$\mathrm{FV} = -\mathrm{PV}(1+r)^n - \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

are to find the number of periods for payments, n, and the interest rate, r.

Excel's Number of Periods Function

$$=\textbf{NPER}(\text{r, PMT, PV, FV, type})$$

This calculates the number of periods required in a given financial problem given all the other parameters. This is a good exercise in rearranging a formula:

Rearrange

$$\text{FV} = -\text{PV}(1+r)^n - \text{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

to find an expression for n.

Excel's Number of Periods Function

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to find an expression for n.

$$n = \frac{1}{\ln(1+r)} \ln\left\{\frac{\frac{\mathrm{PMT}}{r} - \mathrm{FV}}{\frac{\mathrm{PMT}}{r} + \mathrm{PV}}\right\}$$

Note: The chances of you getting an integer out of this expression in a real problem are roughly zero!

Example: How long would it take me to pay off a loan of £10,000 at a rate of 0.5% per month if I can afford to pay £100 per month?

$$\text{PV}= +10000, \quad \text{FV}=0, \quad \text{PMT}= -100, \quad \text{r}= 0.5\%, \quad \text{type}=0$$

We enter:

$$= \textbf{NPER}(0.5\%,\ -100,\ 10000,\ 0,\ 0)$$

and get the answer 138.98.

We have a couple of options:

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2. Pay for 138 but make the last payment bigger.

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1. Pay for 139 periods but make the last payment smaller.
2. Pay for 138 but make the last payment bigger.
3. Pay for 139 periods and pocket the overpayment.

For option 1) calculate

$$= \textbf{FV}(0.5\%,\ 139, -100, 10000, 0)$$

This gives 2.42. I.e., £2.42 should be returned to you at the end, making a final payment of £97.58.

For option 1) calculate

$$= \textbf{FV}(0.5\%,\ 139, -100, 10000, 0)$$

This gives 2.42. I.e., £2.42 should be returned to you at the end, making a final payment of £97.58.

For option 2) calculate

$$= \textbf{FV}(0.5\%,\ 138, -100, 10000, 0)$$

This gives −97.09. I.e., after 138 payments you still owe £97.09, so you pay a final payment of £197.09.

Excel's rate Function

=**RATE**(n, PMT, PV, FV, type, guess)

This calculates the interest rate given all the other parameters.

$$\mathrm{FV} = -\mathrm{PV}(1+r)^n - \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\}$$

Cannot be rearranged to give an equation for r. Excel will have to find r by an iterative method. Microsoft are not telling, but it could be done by solving

$$f(r) = \mathrm{FV} + \mathrm{PV}(1+r)^n + \mathrm{PMT}\left\{\frac{(1+r)^n - 1}{r}\right\} = 0$$

using Newton's method. "guess" is the initial guess for r.

Example: I borrow £1000 over 1 year making payments of £100 per month at the end of each month. What is the monthly interest rate?

We calculate

$$= \textbf{RATE}(12,\ -100,\ 1000,\ 0,\ 0,\ 0)$$

This gives 2.92%.

Further Financial Functions

Excel has many other financial function.

See Dr Bowtell's notes for details on the following:
For calculating how much interest or principal is payed off in any payment period:

=**IPMT**(r, i, n, PV, FV, type)
=**PPMT**(r, i, n, PV, FV, type)

For calculating how much interest or principal is payed off over some longer time (cumulative interest or payment):

cumulative interest function

=**CUMIPMT**(r, n, PV, "start", "end", type)

cumulative principal function

=**CUMPRINC**(r, n, PV, "start", "end", type)