MATHEMATICAL METHODS: COMPLEX VARIABLES 2 COURSEWORK QUESTIONS TO BE HANDED IN BY 4:00PM, MONDAY, 28 JANUARY 2008

- 1. Find *all* the singularities and the corresponding residues of the following functions:
 - (a) $\frac{z^2 + 1}{z^2 1}$ (b) $\frac{\sin z}{(z 1)^5}$ (c) $\frac{\tan z}{z^2}$
- 2. Evaluate the integrals using contour integration techniques

(a)
$$\int_0^{2\pi} \frac{\sin\theta}{5+4\sin\theta} d\theta$$
 (b) $\int_{-\infty}^{\infty} \frac{1+2x^2}{x^4+10x^2+9} dx$

3. Evaluate the following infinite series:

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

4. The function f(x) is periodic with period p. Write down its general Fourier expansion along with integral expressions for the Fourier coefficients a_0 , a_n and b_n .

The periodic function f(x) with period 2 is defined in the interval 0 < x < 1 by

$$f(x) = x^2$$

Sketch this function for $-3 \le x \le 3$ for the cases (i) where f(x) is even, and (ii) f(x) is odd.

In each of the two above cases what can you deduce about the Fourier coefficients?

Calculate the Fourier coefficients for the function f(x) when it is *odd*. Write down its Fourier expansion.