Mathematical Methods: Complex Variable

Sets of Points in the Complex Plane

- **metric** or **distance** between z_1 and z_2 is $|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
- A **(circular) neighbourhood** or **delta/δ neighbourhood** of a point z_0 is the set $\{z : |z - z_o| < \delta\}$ for some δ (often small).
- A **deleted δ neighbourhood** of a point z_0 is a neighbourhood of z_0 excluding the point z_0 itself.
- The **limit point** or **cluster point** or **point of accumulation** z_0 of a set is such that every neighbourhood of the point z_0 contains points of the set.
- An **interior point** z_0 of a set is such that some neighbourhood of it consists only of points of the set.
- A **boundary point** z_0 of a set is a point such that every neighbourhood of z_o contains points belonging to the set *and* points not belonging to the set.
- An **exterior point** is a point that is neither an interior point nor a boundary point.
- A set is **open** if and only if all its points are interior points.
- A set is **closed** if and only if it contains all its accumulation points.

  Note: Sets may be both open and closed (e.g. empty set $\emptyset$, or whole complex plane) or neither (e.g. $1 < |z| \leq 2$).
- A set is **connected** if *any* two points can be joined by a path consisting of finitely many straight line segments all points of which lie in the set.
- An open connected set is called a **open domain** or just **domain**.
- A **bounded** set lies in $|z| < K$ for some K.

Two other important theorems which we may or may not require, but which you should be aware of:

- **Bolzano-Weierstrass Theorem** (or Weierstrass-Bolzano Theorem). Every bounded infinite set has at least one accumulation point.
- **Heine-Borel Theorem**. If any closed and bounded set (i.e. a **compact** set) is covered by a collection of open sets, then it can be covered by a finite sub-collection.