Mathematical Methods Coursework
Complex Variables 1
To be handed in on Tuesday 27 November 2007, C123

1. Find whether the following functions are harmonic (satisfy Laplace's equation). In each case where it is harmonic find the conjugate harmonic function, and ence find the corrsponding analytic function of which it is the real part.

   (a) $y^2 + x^2$, (b) $y^2 - x^2$,

   (c) $\sin 2x \sinh y$ (d) $\cosh x \cos y$,

   (e) $x^3 y - x y^3$, (f) $\tan^{-1}(x/y)$.

2. Show that if an analytic function, $f(z)$, has constant modulus for all z, then $f(z)$ is a constant. [Hint: consider the variation of $|f(z)|^2 = u(x,y)^2 + v(x,y)^2$, and use the Cauchy-Riemann equations.]

3. Which of the following series converge, giving your reasons

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{n+2}{3n+2i}$ (b) $\displaystyle\sum_{n=0}^{\infty} n\left(\frac{1+i}{2}\right)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2n}}{n!}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{i^n}{n}$ [You may not find this one obvious]

4. Find *all* the Taylor and Laurent Series of the following functions around the indicated centres:

   (a) $\dfrac{2}{1-z^2}$, $z_0 = 0$ (b) $\dfrac{1}{z^3+6z^2+11z+6}$, $z_0 = 0$

   (c) $\dfrac{e^{-z}}{z-1}$, $z_0 = 1$ (d) $\dfrac{1}{z^3+1}$, $z_0 = -1$