Mathematical Methods: Fourier Series

Fourier Series: First Example

Example: Sketch the periodic function with period 2π given by

$$f(x) = \begin{cases} -1 & -\pi < x \le 0 \\ +1 & 0 < x \le \pi \end{cases}$$

Find its Fourier series.

Fourier Series: square wave

Here are the first few approximations to the square wave using Fourier series that we found. The square wave was given by

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ +1 & 0 < x < \pi \end{cases}$$

with

$$f(x+2\pi) = f(x)$$
 for all x .

The Fourier series for this function is

$$f(x) = \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x + \cdots$$

We can make approximations to the original function by truncating the series after a given number of terms. The more terms that are included the more accurate the approximation.

The approximation using only the $\sin x$ term is



The approximation using up to the $\sin 3x$ term is



The approximation using up to the $\sin 5x$ term is



The approximation using up to the $\sin 7x$ term is



The approximation using up to the $\sin 9x$ term is



The approximation using up to the $\sin 21x$ term is



The approximation using up to the $\sin 101x$ term is



Note: There is always an overshoot in the approximations when there is a discontinuity in the original function. This slows down the convergence to the original function.