

FOURIER SERIES

Below are the first few approximations to the square wave using Fourier series that were found in the lectures. The square wave was given by

$$f(x) = \begin{cases} -1 & -\pi < x < 0 \\ +1 & 0 < x < \pi \end{cases}$$

with

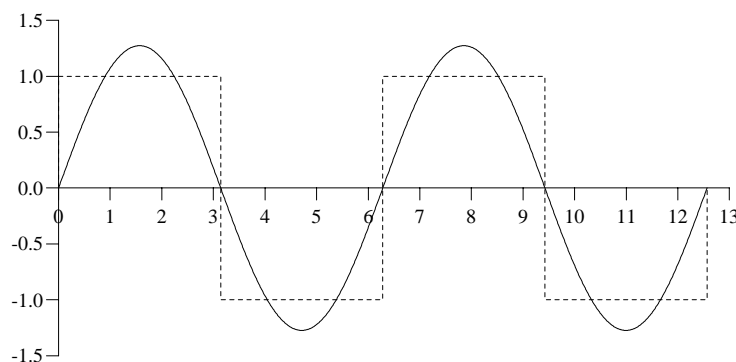
$$f(x + 2\pi) = f(x) \quad \text{for all } x.$$

The Fourier series for this function is

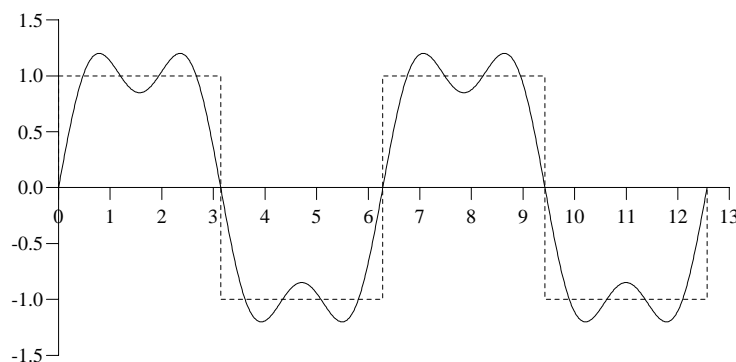
$$f(x) = \frac{4}{\pi} \sin x + \frac{4}{3\pi} \sin 3x + \frac{4}{5\pi} \sin 5x + \cdots$$

We can make approximations to the original function by truncating the series after a given number of terms. The more terms that are included the more accurate the approximation.

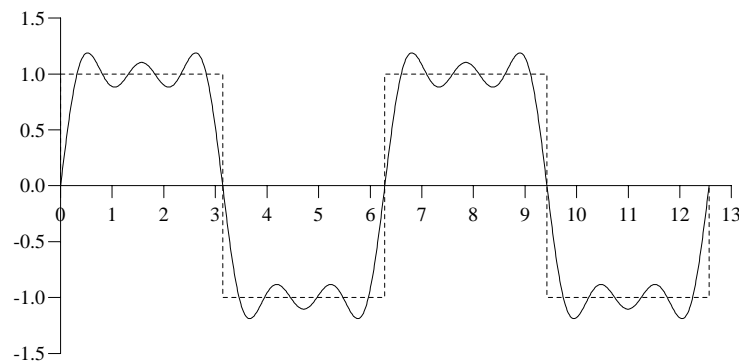
The approximation using only the $\sin x$ term is



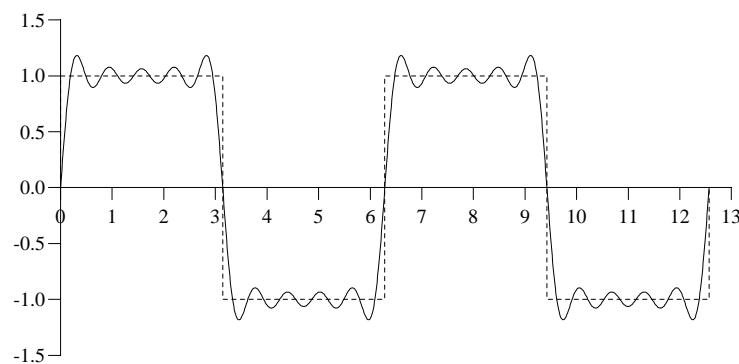
The approximation using up to the $\sin 3x$ term is



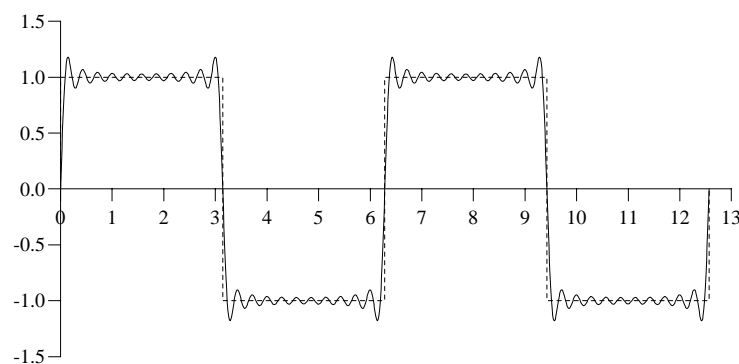
The approximation using up to the $\sin 5x$ term is



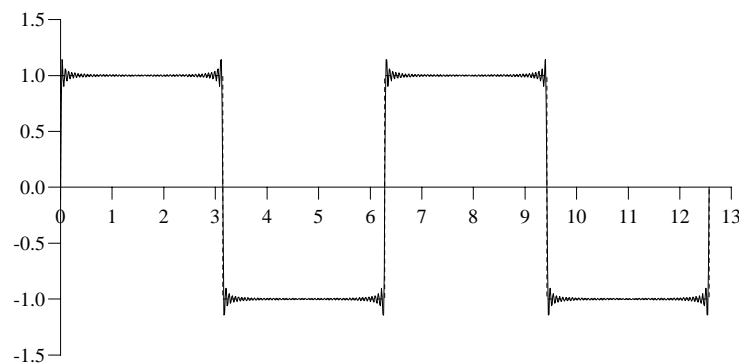
The approximation using up to the $\sin 9x$ term is



The approximation using up to the $\sin 21x$ term is



The approximation using up to the $\sin 101x$ term is



Note: There is always an overshoot in the approximations when there is a discontinuity in the original function. This slows down the convergence to the original function.