



CITY UNIVERSITY
LONDON

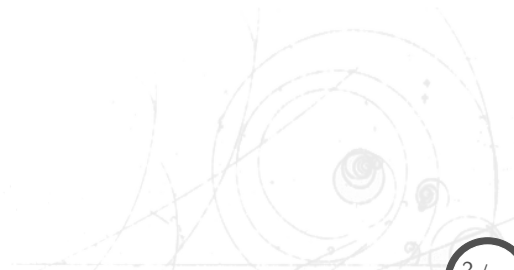
What do mathematical physicists do? An example: A generalised version of Heisenberg's uncertainty relation

Andreas Fring

Ark Academy, 9th of March, 2022

Outline:

1) Classical Physics



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- 2) Heisenberg's uncertainty relation in quantum mechanics

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What do *classical* physicists do?

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Measure what is measurable, and make
measurable what is not so.

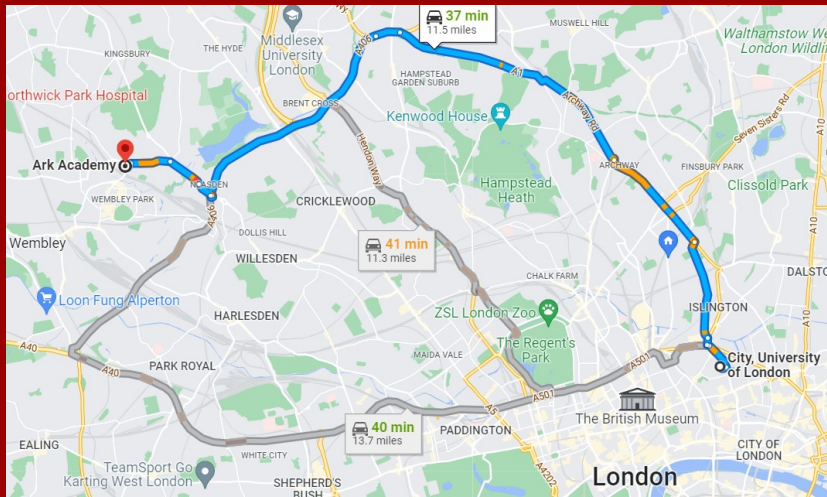
(Galileo Galilei)

izquotes.com

What do *classical* physicists measure?

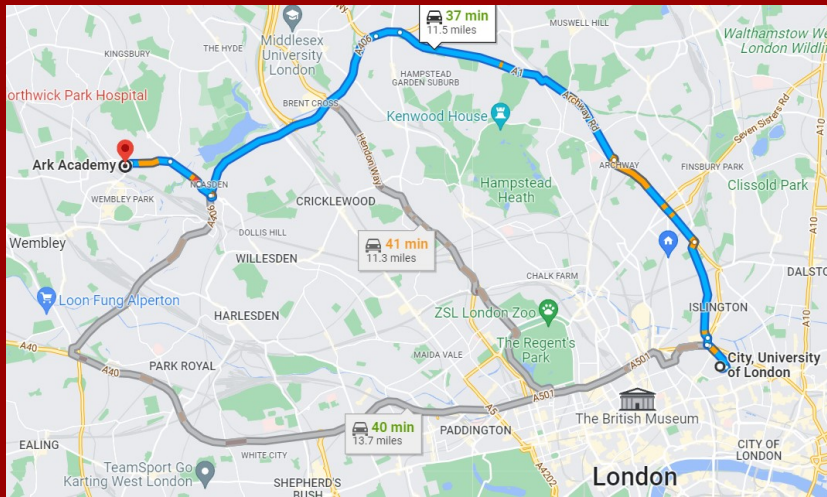
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For instance: distances



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1795:



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1983: The metre is the length of the path travelled by light in vacuum during a time interval of $1/299\,792\,458$ of a second.

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More examples of measurable quantities:

Length (meter,m)

Time (second,s)

Mass (kilogram,kg)

Temperature (kelvin,K)

Electric current (ampere,A)

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Velocity (m/s)

Momentum (kg m/s)

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The speed of a falling object depends on its weight.

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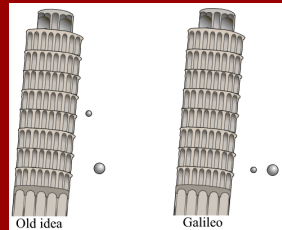
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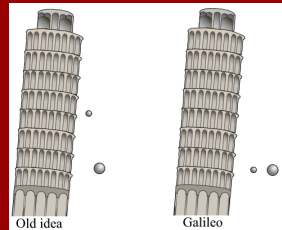
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⇒ Aristotle was wrong

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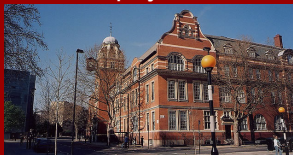
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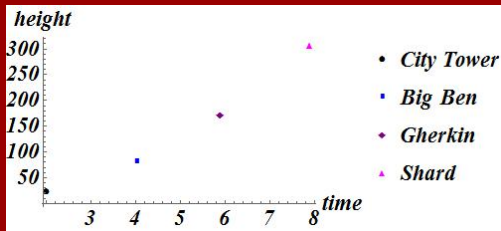
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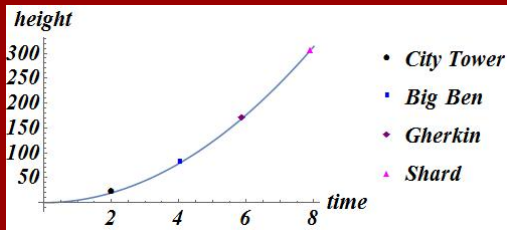
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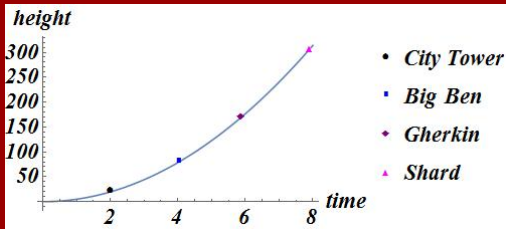
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gravitational acceleration: $g = 9.81 \frac{m}{s^2}$

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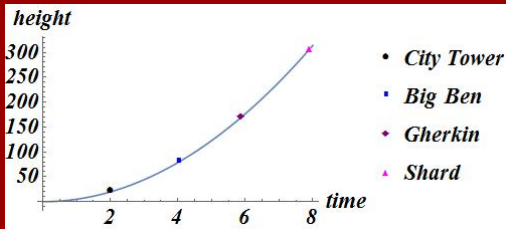
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$$\sqrt{2 \times 828m / 9.81 \frac{s^2}{m}} = 12.99s$$



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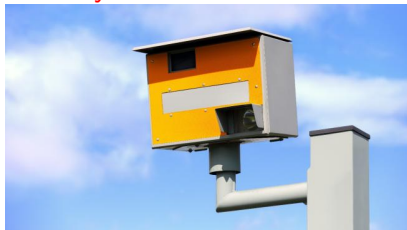
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Heisenberg: "What we observe is not nature in itself but nature exposed to our method of questioning."

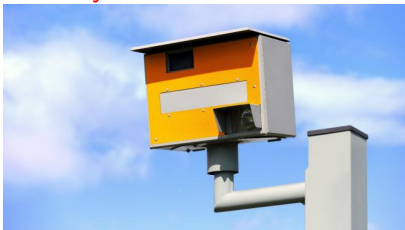
Measuring two quantities at the same time

Velocity:




Measuring two quantities at the same time

Velocity:



Location:

 **POLICE SCOTLAND**
Keeping people safe

Date of Issue: 18 December 2013
Our Reference: 8AG4513/007

Dear Sir/Madam

CONDITIONAL OFFER OF FIXED PENALTY NOTICE
VEHICLE REGISTRATION MARK: CY08AZV

I refer to previous communications whereby you have admitted being the driver of a vehicle detected speeding as detailed below:-

Offence Details:	Exceed 60 mph Speed Limit - Class of Vehicle
Date & Time:	10-Nov-13 at 10:22 hours
Vehicle:	CY08AZV VOLKSWAGEN
Vehicle Speed:	82 mph
Offence Location:	A52 CROSSGATES TO COWDENBEATH ROAD

The Road Traffic Offenders Act 1988 Section 75 as amended, permits that this offence may be dealt with by way of a Conditional Offer of Fixed Penalty Notice. This allows for the matter to be concluded by payment of a fixed penalty of £100 and an endorsement of your licence with 3 penalty points, at which time any liability for conviction of the offence is discharged. Acceptance of the Conditional Offer of Fixed Penalty is final and no discussions or review of the facts of this case can thereafter take place. Should you fail to accept this Conditional Offer of Fixed Penalty then a report detailing the full circumstances will be submitted to the Procurator Fiscal for consideration of a prosecution.

If you wish to accept the Conditional Offer of Fixed Penalty you must present £100 together with your driving licence and paper counterpart (if applicable) to the Scottish Court Service within 28 days of the date of issue as shown above, during which time you cannot be prosecuted. Further information is provided in the 'How to Pay' section overleaf.

DO NOT SEND YOUR PAYMENT OR DRIVING LICENCE TO THE ADDRESS ABOVE

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- mistake in measuring velocity: Δv
- mistake in measuring the location: Δx

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Driver wins speeding ticket battle after proving that the road markings used by camera to indicate speed were the wrong distance apart

- David Erasmus, 55, was sent a ticket for allegedly speeding past a primary
- He noticed markings used to work out speed were three inches too short
- His case was formally dismissed after a trial at Llanelli Magistrates' Court
- Decision means other drivers may be able to appeal their convictions

By HANNAH PARRY FOR MAILONLINE

PUBLISHED: 18:10, 12 December 2014 | UPDATED: 00:52, 13 December 2014

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In classical physics we can achieve $\Delta v \rightarrow 0$ and $\Delta x \rightarrow 0$.
It just depending on the precision of our instruments.

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In quantum mechanics we can no longer measure certain quantities simultaneously:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

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\hbar : reduced Planck constant, that is a constant of nature (like g)

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In turn, demanding to know exactly the speed of the particle implies that we can not know its location.

This is a fundamental property of nature and does not depend on our instruments!

Where does this come from?

De Broglie: Every object in the universe is a wave.

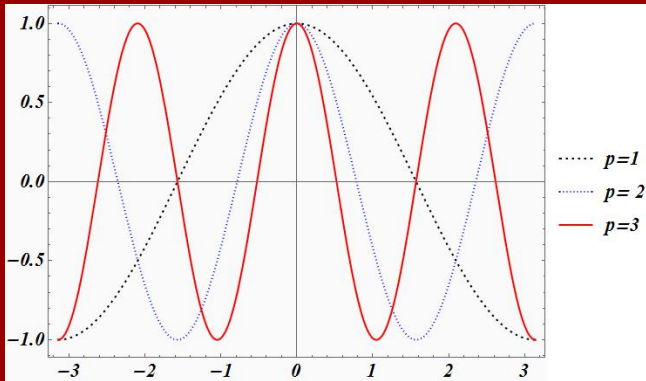
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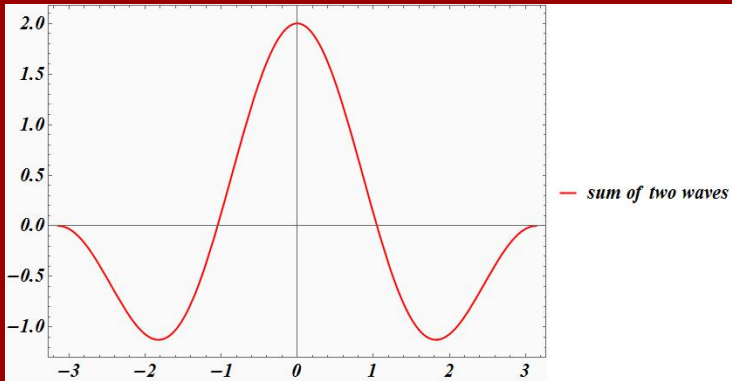
Waves with different momenta



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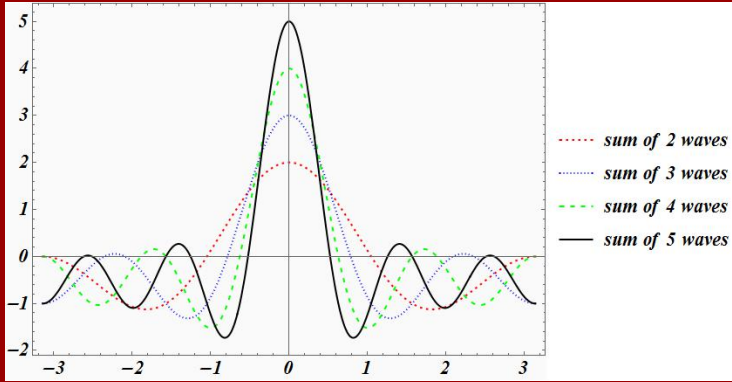
Sum of waves with momentum 1 and 2



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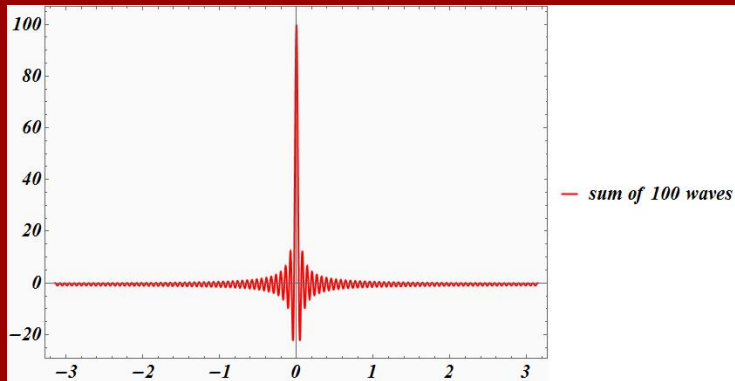
Sum of several waves



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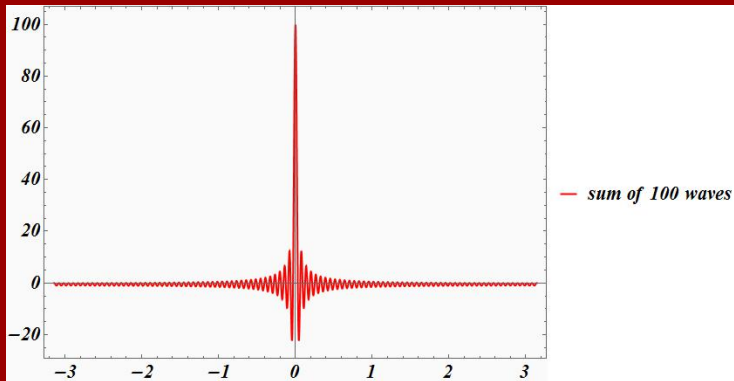
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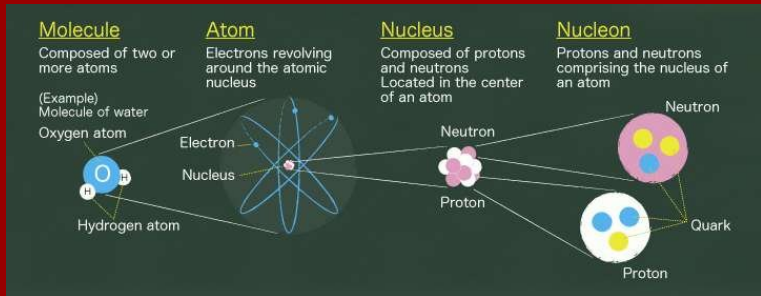
Sum of 100 waves



Now we have a good localisation, but have used 100 different momenta to achieve this.

Classical particles

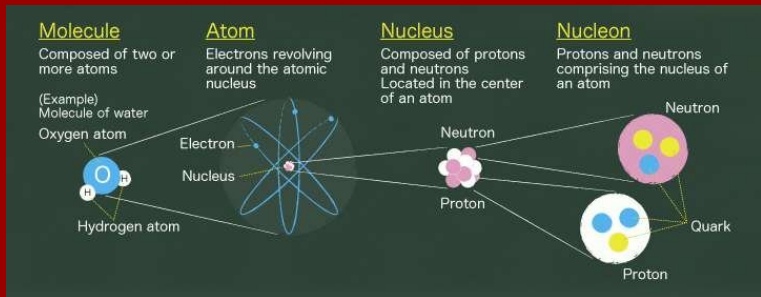
Classical particles



Concept and key properties:

- localised objects (lumps)

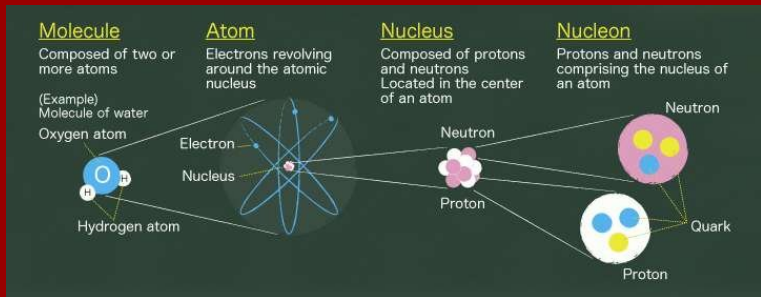
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Concept and key properties:

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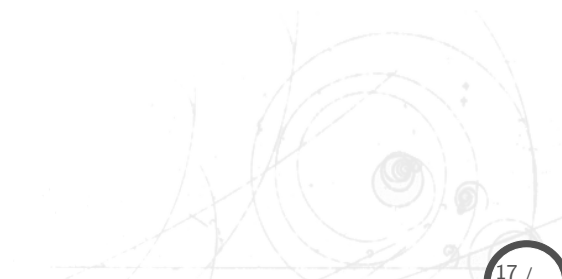
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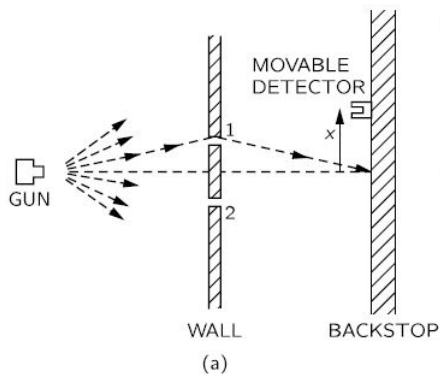
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- no interference
- intensity is 0 or 1

An interference experiment with classical particles

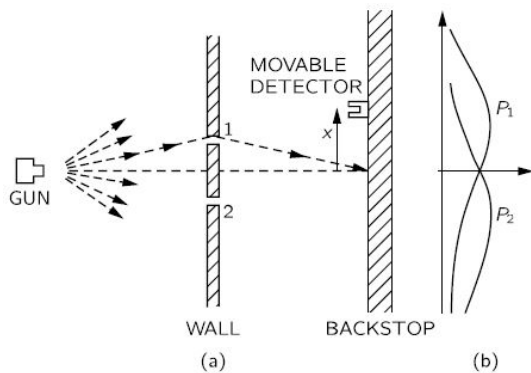


An interference experiment with classical particles



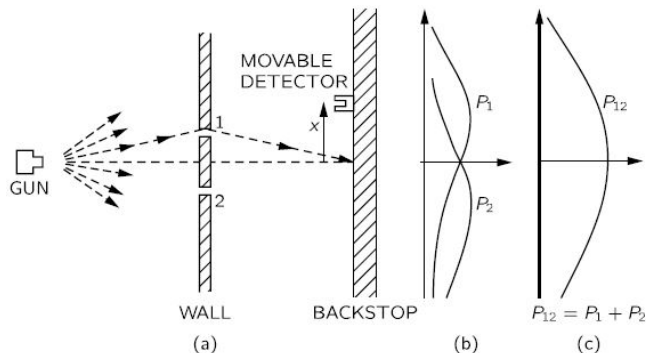
a) bullets go through slit 1 or 2 and are detected at the backstop

An interference experiment with classical particles



- a) bullets go through slit 1 or 2 and are detected at the backstop
- b) bullets go through slit 1 $\Rightarrow P_1$ or (2 $\Rightarrow P_2$)

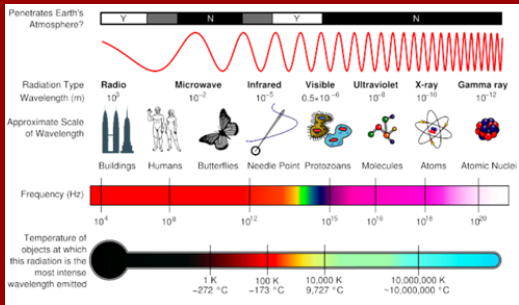
An interference experiment with classical particles



- a) bullets go through slit 1 or 2 and are detected at the backstop
- b) bullets go through slit 1 $\Rightarrow P_1$ or (2 $\Rightarrow P_2$)
- c) bullets go through slit 1 or 2 $\Rightarrow P_{12} = P_1 + P_2$

Classical waves

Classical waves



Classical waves



Concept and key properties:

Classical waves



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Classical waves



Concept and key properties:

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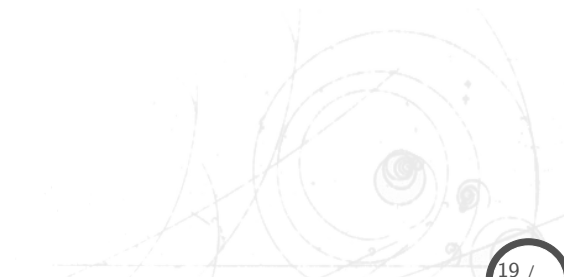
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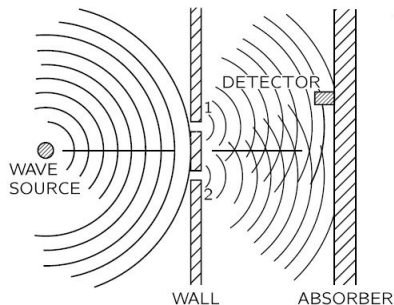
Concept and key properties:

- nonlocalised objects
- waves interfere
- intensity can take any value

An interference experiment with water waves



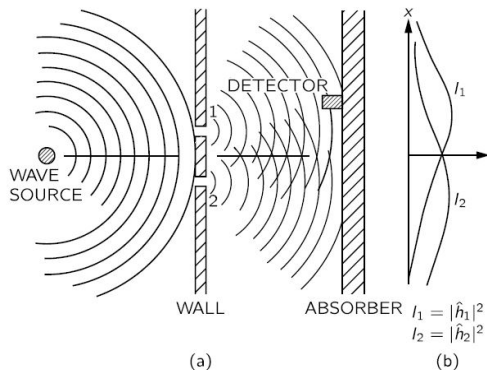
An interference experiment with water waves



(a)

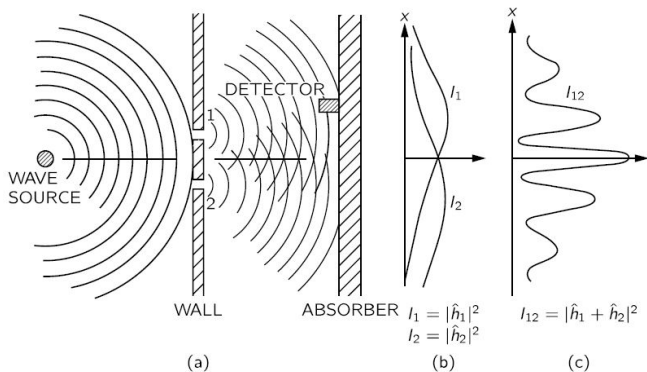
a) wave goes through slit 1 or 2, height $h_{1/2}$ detected at absorber

An interference experiment with water waves



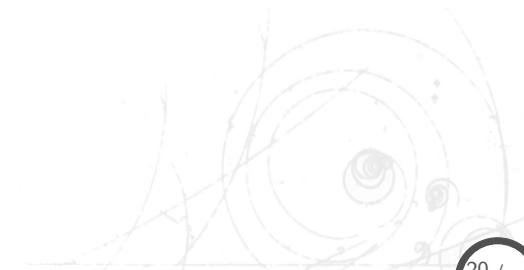
- a) wave goes through slit 1 or 2, height $h_{1/2}$ detected at absorber
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An interference experiment with water waves

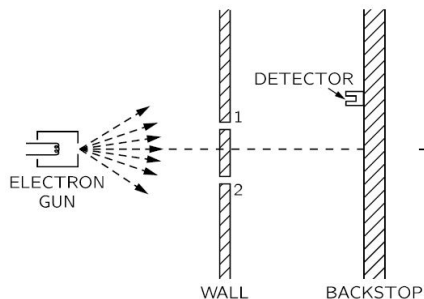


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- b) wave goes through slit 1 $\Rightarrow I_1 = |h_1|^2$ or (2 $\Rightarrow I_2 = |h_2|^2$)
- c) wave goes through slit 1 or 2 $\Rightarrow I_{12} = |h_1 + h_2|^2 \neq I_1 + I_2$

An interference experiment with electrons (e^-)



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(a)

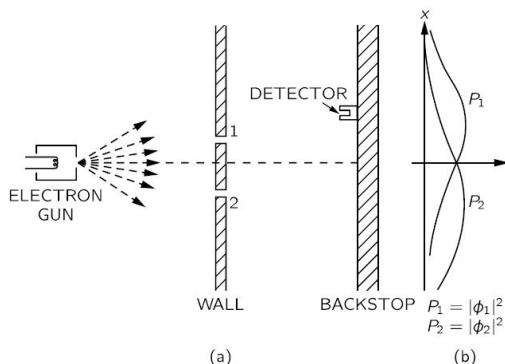
a) always hear the same clicks in detector, e^- arrive in lumps

An interference experiment with electrons (e^-)

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Proposition: Each e^- passes *either* through slit 1 *or* slit 2

An interference experiment with electrons (e^-)

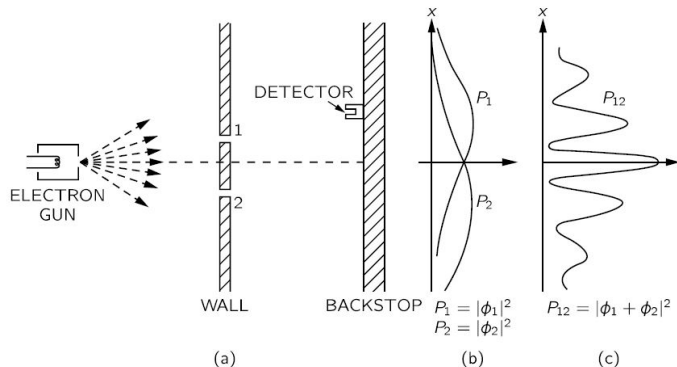


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Proposition: Each e^- passes *either* through slit 1 or slit 2

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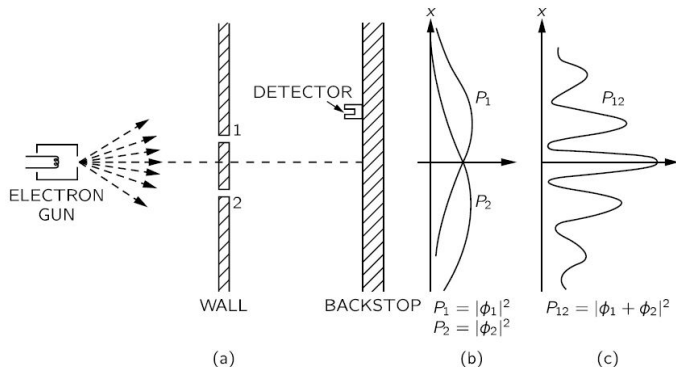
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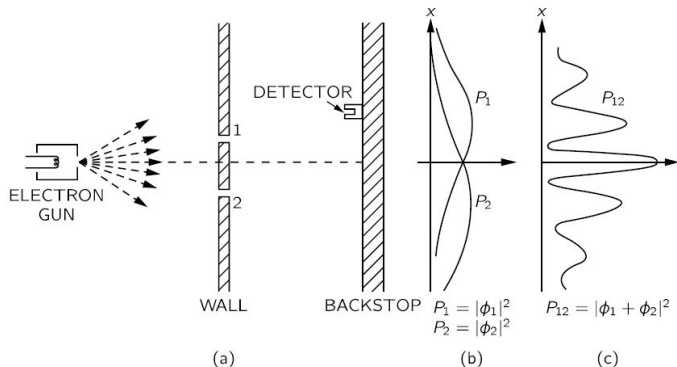
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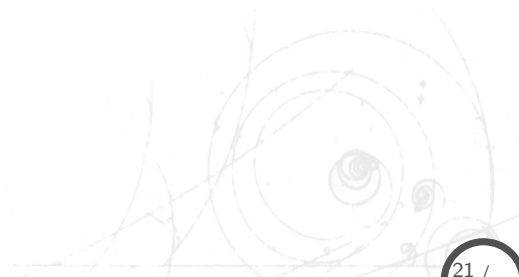
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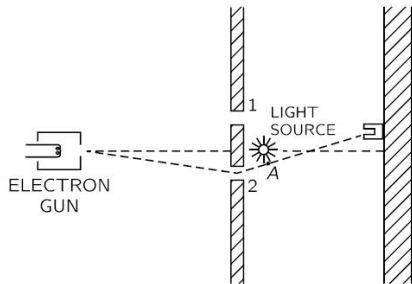
\Rightarrow The proposition must be false.



Watching the e^-



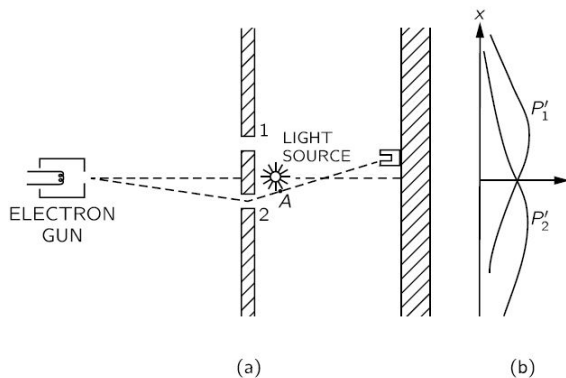
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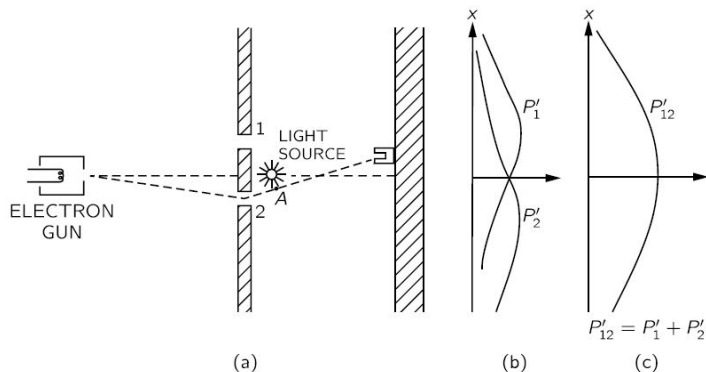
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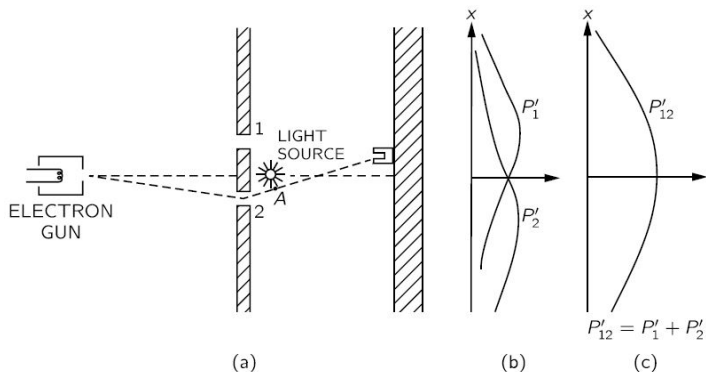
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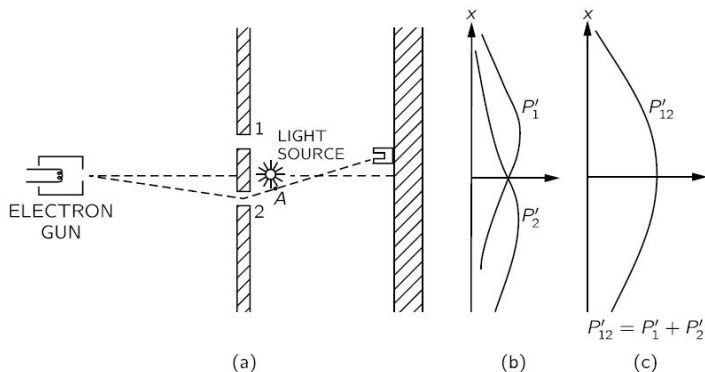
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when we watch e^- it behaves like a particle.

\Rightarrow The proposition seems to be correct.



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Heisenberg's uncertainty relation:

It is not possible to design an experiment so that we know the position of the object without disturbing it.

Modern variations of Heisenberg's uncertainty relation:

We change now the right hand side of the inequality:

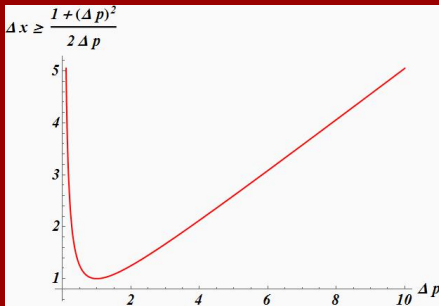
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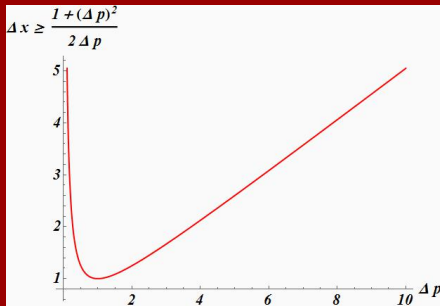


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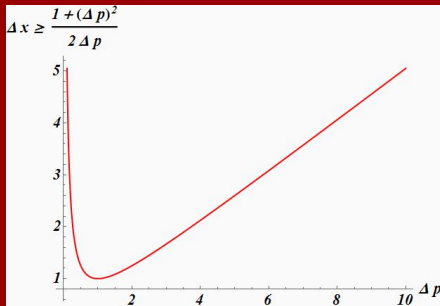
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We can never achieve $\Delta x = 0$, even when $\Delta p \rightarrow \infty$.
Below $\Delta x_{\min} = 0$ we can not know anything.

If we knew what we were doing, it would not be called research, would it?

Albert Einstein

It is a capital mistake to theorize before one has data.

Sherlock Holmes

It is a good thing for a research scientist to discard a pet hypothesis every day before breakfast.

Konrad Lorenz

Nothing has such power to broaden the mind as the ability to investigate systematically and truly all that comes under thy observation in life.

Marcus Aurelius

Research is creating new knowledge.

Neil Armstrong



