ME 1110 – Engineering Practice 1

Engineering Drawing and Design - Lecture 15

Mechanical Elements – Shafts

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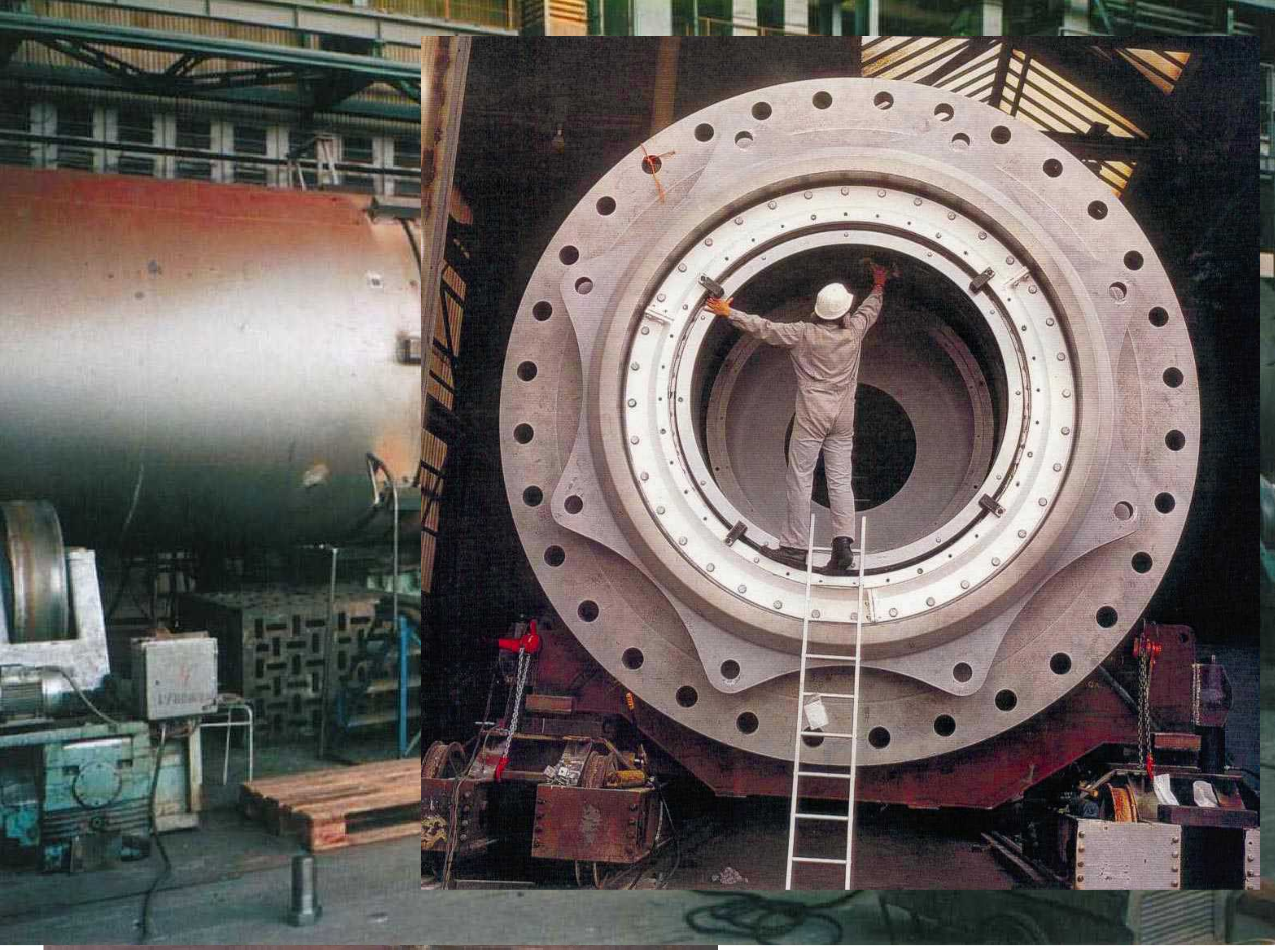
www.staff.city.ac.uk/~ra600/intro.htm

Introduction

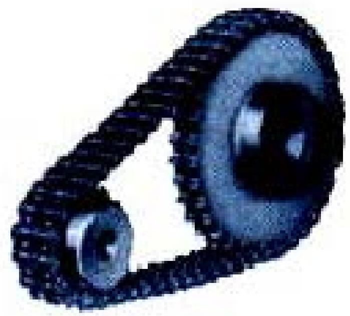
- ***Shaft*** – a rotating element used to transmit power or motion. It provides axis of rotation for rotating elements and controls their motion.
- ***Axle*** – a non-rotating element that carries no torque and is used to support rotating elements.
- ***Spindle*** - a short shaft
- There are two aspects of shaft design:
  - » Deflection and rigidity (bending and torsional deflection)
  - » Stress and strength
- To design a shaft, other elements: gears, pulleys, bearings … should be located and specified.
- Design objective <u>necessary to check if a shaft diameter is sufficient to sustain loads</u>



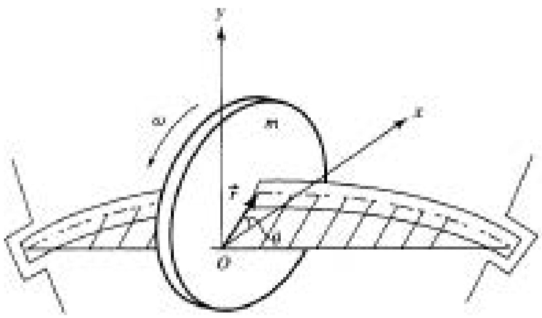
Common shaft loading mechanisms

Spur Gears

Chain Drives



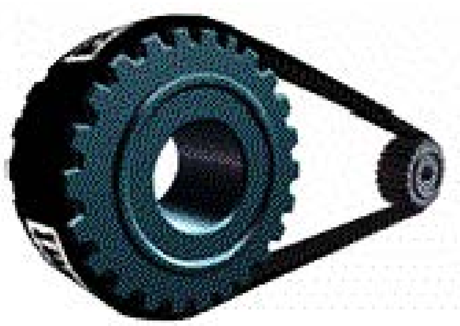
Unbalanced Mass



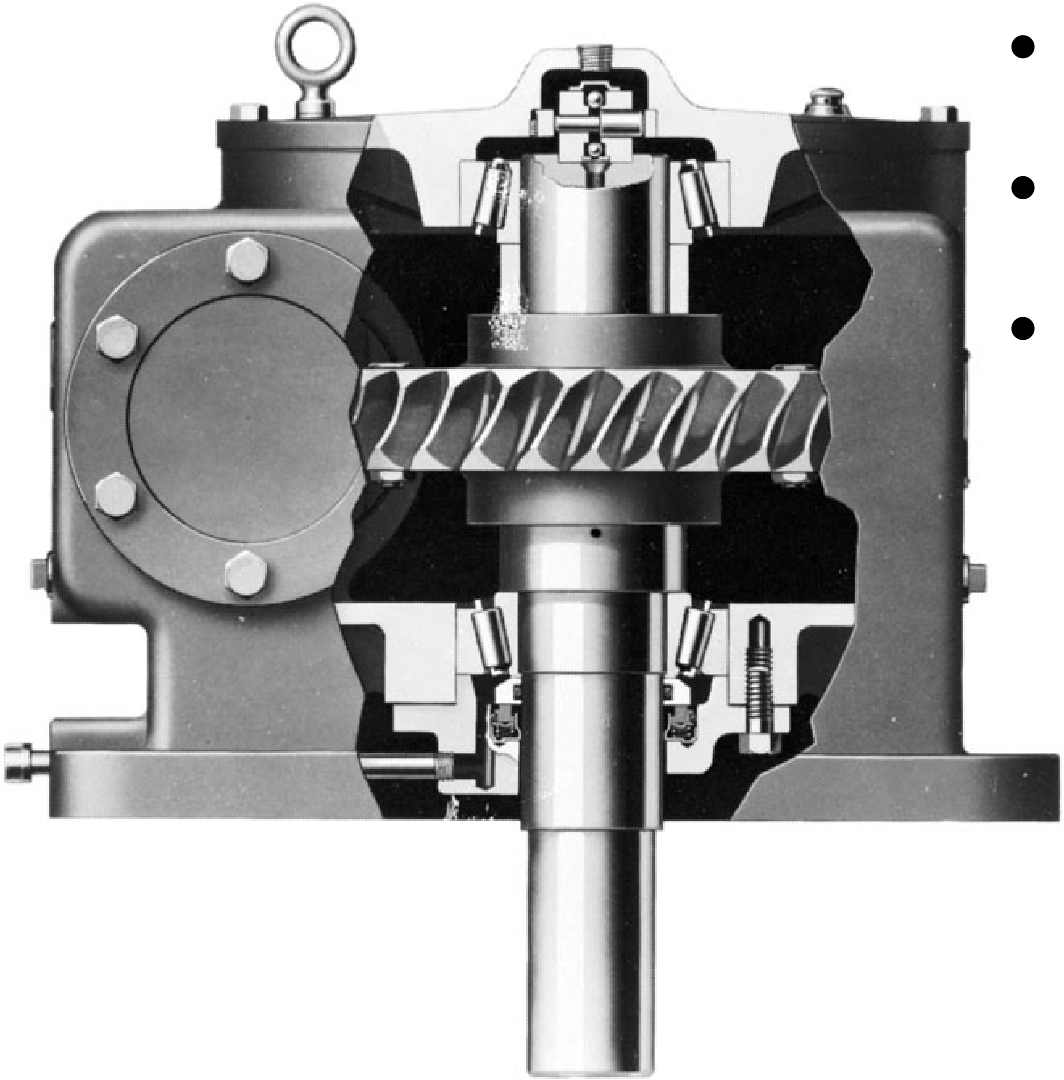
Helical Gears

Spiral Bevel Gears

Belt Drives



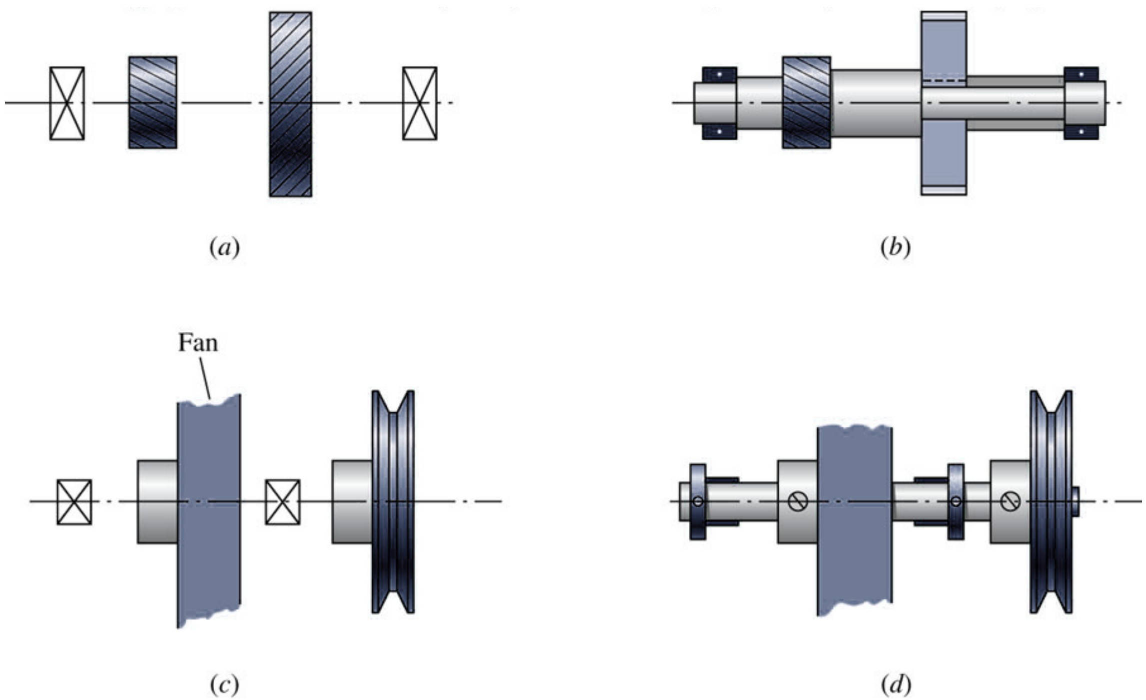
Shaft design characteristics



- Shaft is usually of circular cross section
- Deflections are function of the geometry and load.
- Steps in the shaft design are:
  - » Define shaft topology
  - » Specify driving elements
  - » Free body diagram
  - » Select bearings
  - » Consider shaft deflection and stress
  - » Specify connections
  - » Dimensions

Shaft design

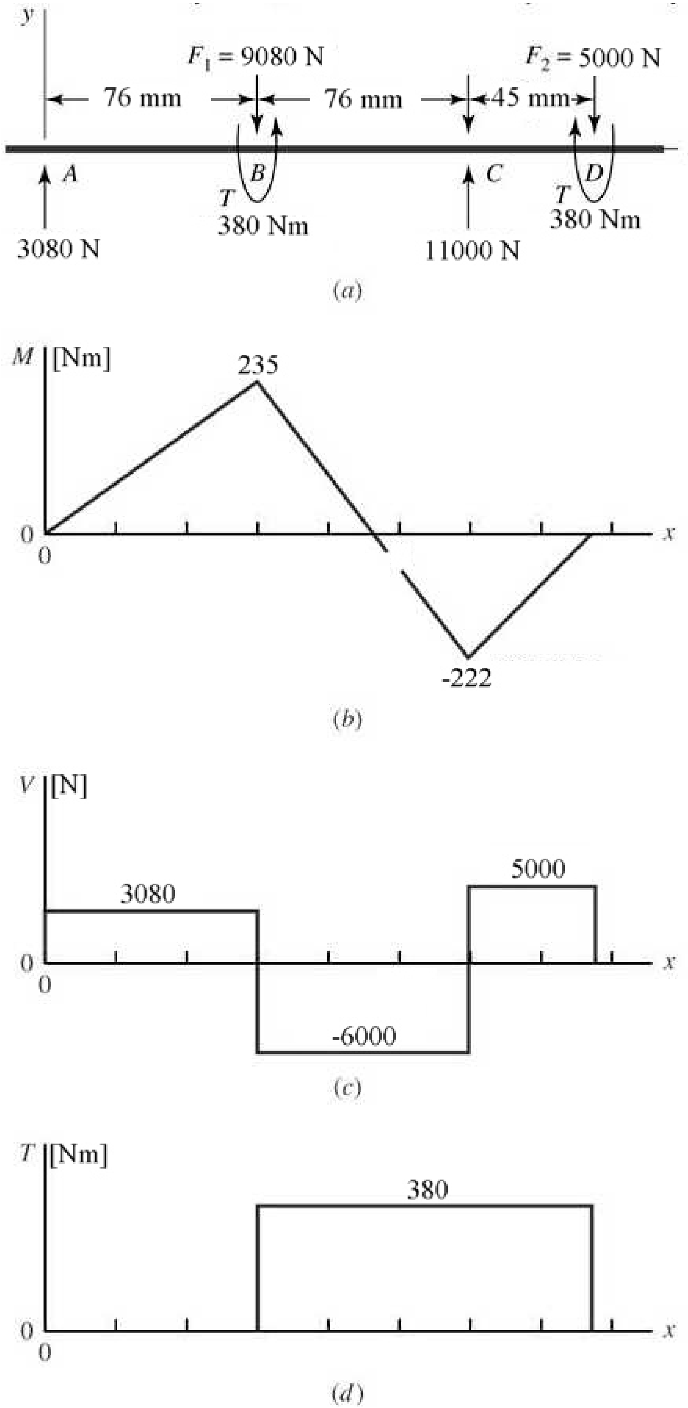
- ***Shaft topology***
  a) Choose a shaft configuration to support and locate the two gears + two bearings
  b) Solution uses an integral pinion, shaft with three shoulders, key, keyway and sleeve. Bearings are located in the housing
  c) Choose fan shaft configuration
  d) Solution uses sleeve bearings, a straight through shaft, locating collars, setscrews, pulley and fan



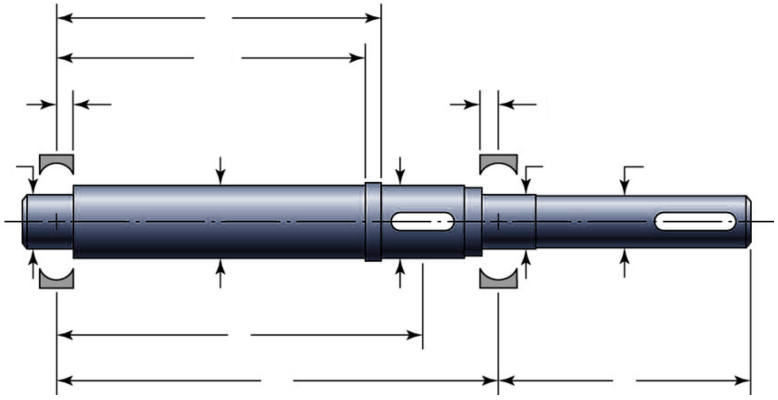
- ***Driving elements***

1. Driving elements (gears, pulleys, sprockets …) have to be selected and calculated
2. Minimum diameter of a rotating element and forces acting on it are relevant for a shaft design

Shaft design

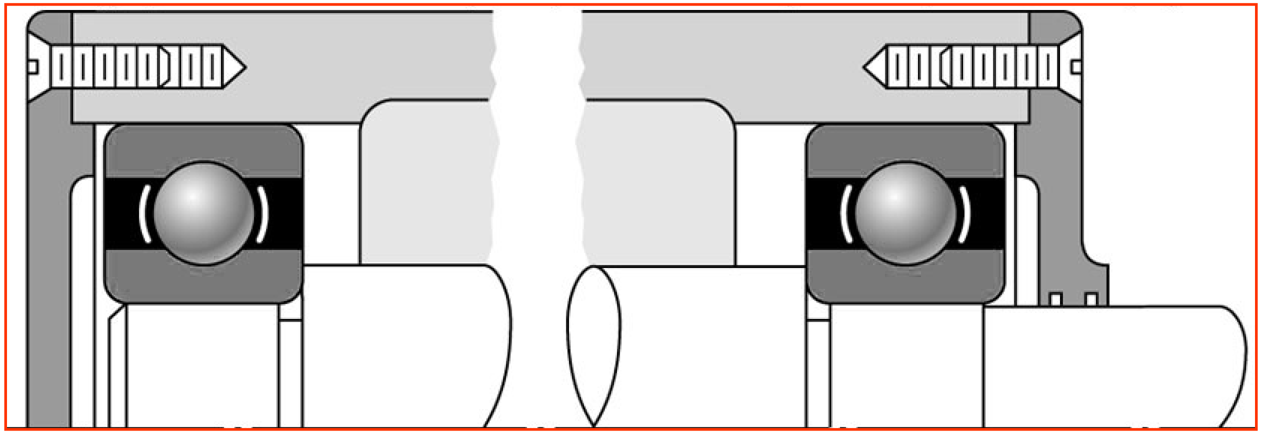


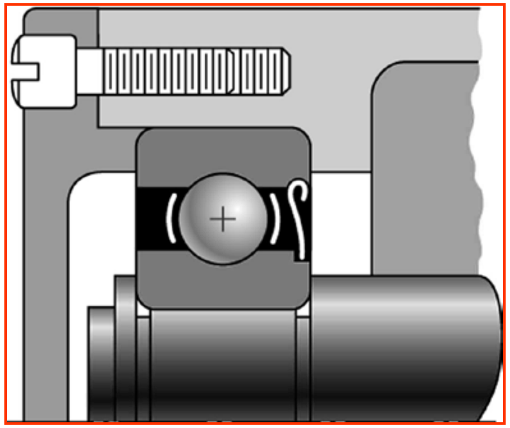
- ***Free body diagram***
  Free body diagram is calculated such that the system of interest is separated from the surrounding and connections are replaced by forces
  a) Reactions in bearings & force diagram
  b) Bending moment
  c) Torsional moment ($P=\omega T$)
- ***Bearing selection***
  a) Equivalent load (forces)
  b) Bearing rating life based on the size
  c) Positioning and lubrication

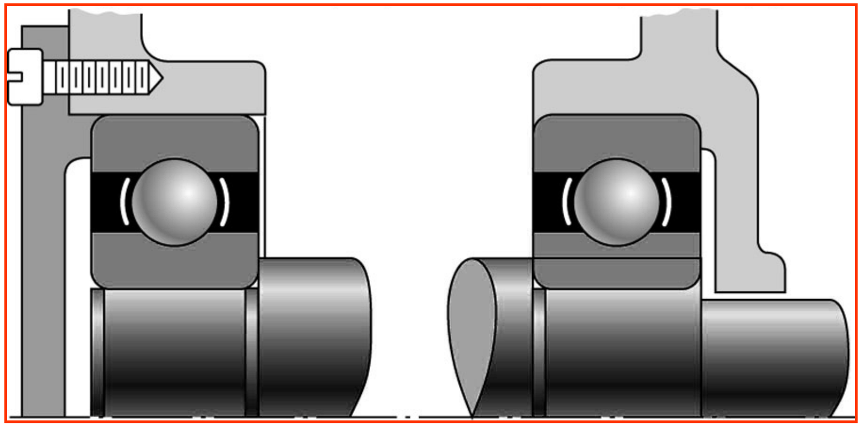


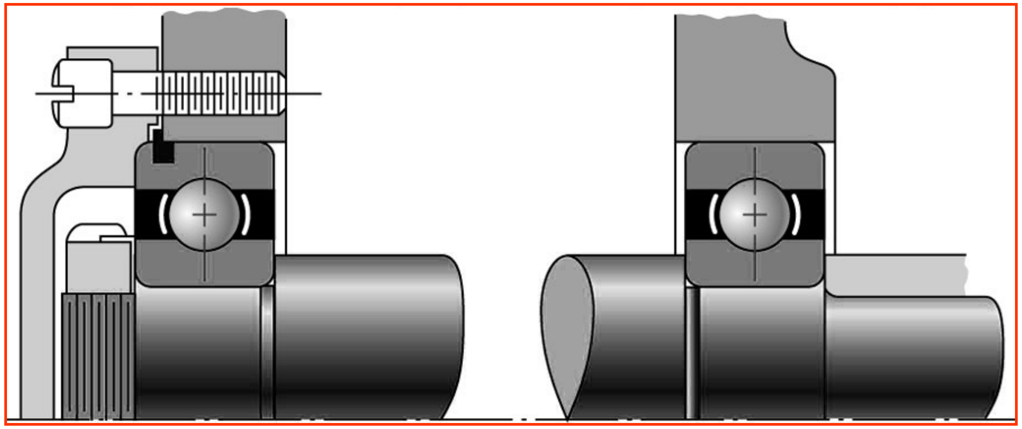
Design web

Shaft design – bearing positioning

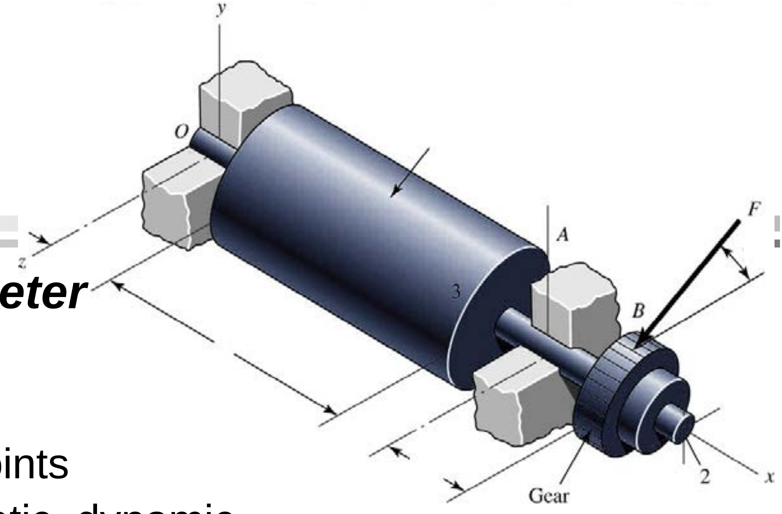








Shaft design



- ***Shaft deflection and stress – minimum diameter***
  Difficult to calculate exactly. Reasons for complexity:
  a) Variable shaft diameter
  b) Undercuts and grooves – stress concentration points
  c) Type of load – axial, radial, torsional, bending, static, dynamic …

- In this course we will calculate a minimum shaft diameter without considering stress concentration points.
- Calculations will be based on the maximum static load.
- Diameter will be estimated for allowable stress which depends on the shaft material.

Bending stress

$$\sigma_z = \frac{M}{Z} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$$

Torsional stress

$$\tau_{zy} = \frac{T}{S} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$$

Minimum diameter
distortion energy theory

$$d = \sqrt[3]{\frac{32 f_s}{\pi S_y} \sqrt{M^2 + \frac{3}{4} T^2}}$$

$c=d/2$	- maximum span
$I=\pi d^4/64$	- second moment of area
$Z=c/I$	- section modulus
$J=\pi d^4/32$	- second polar moment of area
$S=c/J$	- polar section modulus
f_s	- factor of safety

Tensile Properties of Some Metals

Material	Ultimate Strength, S_u		Yield Strength, S_y		σ_0 [a]		m [a]	ϵ_{Tf} [a]
	ksi	MPa	ksi	MPa	ksi	MPa		
Carbon and alloy steels								
1002 A[b]	42	290	19	131	78	538	0.27	1.25
1010 A	44	303	29	200	82	565	0.23	1.20
1018 A	49.5	341	32	221	90	621	0.25	1.05
1020 HR	66	455	42	290	115	793	0.22	0.92
1045 HR	92.5	638	60	414	140	965	0.14	0.58
1212 HR	61.5	424	28	193	110	758	0.24	0.85
4340 HR	151	1041	132	910	210	1448	0.09	0.45
52100 A	167	1151	131	903	210	1448	0.07	0.40
Stainless steels								
302 A	92	634	34	234	210	1448	0.48	1.20
303 A	87	600	35	241	205	1413	0.51	1.16
304 A	83	572	40	276	185	1276	0.45	1.67
440C A	117	807	67	462	180	1241	0.14	0.12
Aluminum alloys								
1100-0	12	83	4.5	31	22	152	0.25	2.30
2024-T4	65	448	43	296	100	690	0.15	0.18
7075-0	34	234	14.3	99	61	421	0.22	0.53
7075-T6	86	593	78	538	128	883	0.13	0.18
Magnesium alloys								
HK31XA-0	25.5	176	19	131	49.5	341	0.22	0.33
HK31XA-H24	36.2	250	31	214	48	331	0.08	0.20
Copper alloys								
90-10 Brass A	36.4	251	8.4	58	83	572	0.46	—
80-20 Brass A	35.8	247	7.2	50	84	579	0.48	—
70-30 Brass A	44	303	10.5	72	105	724	0.52	1.55
Naval Brass A	54.5	376	17	117	125	862	0.48	1.00

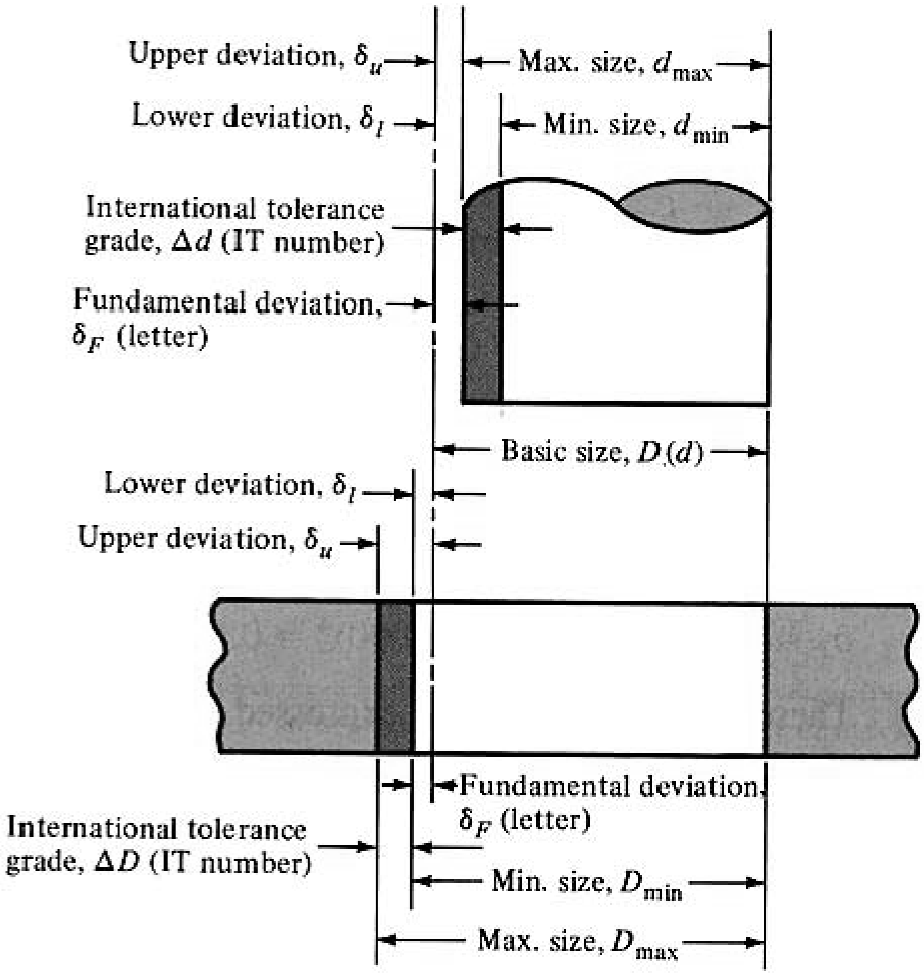
Shaft diameter vs Torque

Shaft Dia	Pure Torque	Power ($P=\omega T$) (at 100 rpm)
mm	Nm	kW
30	132	1.4
40	313	3.3
50	612	6.4
60	1058	10.6
75	2068	21.6
80	2510	26
100	4900	51.3

How to connect elements to the shaft?

- Interference fits
- Keys & Keyseats
- Pins
- Hubs
- Integral shaft
- Splines

Limits and Fits



$D \equiv$ basic size of hole

$d \equiv$ basic size of shaft

$\delta_u \equiv$ upper deviation

$\delta_l \equiv$ lower deviation

$\delta_F \equiv$ fundamental deviation

$\Delta D \equiv$ tolerance grade for hole

$\Delta d \equiv$ tolerance grade for shaft

Tolerance difference between the maximum and minimum size limits of a part.

International Tolerance Grade Numbers are used to specify the size of a tolerance zone.

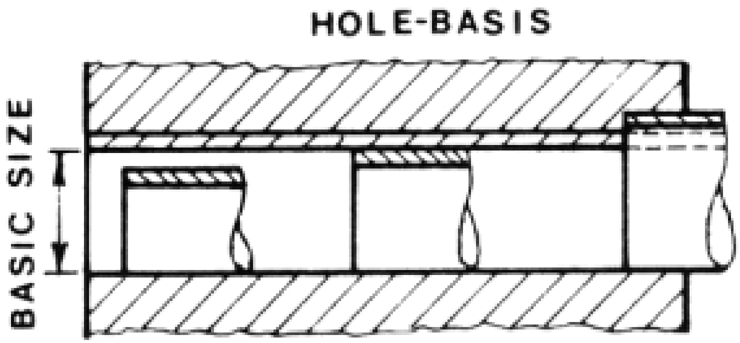
International tolerance grade numbers

BASIC SIZES	TOLERANCE GRADES IT6	IT7	IT8	IT9	IT10	IT11
0–3	0.006	0.010	0.014	0.025	0.040	0.060
3–6	0.008	0.012	0.018	0.030	0.048	0.075
6–10	0.009	0.015	0.022	0.036	0.058	0.090
10–18	0.011	0.018	0.027	0.043	0.070	0.110
18–30	0.013	0.021	0.033	0.052	0.084	0.130
30–50	0.016	0.025	0.039	0.062	0.100	0.160
50–80	0.019	0.030	0.046	0.074	0.120	0.190
80–120	0.022	0.035	0.054	0.087	0.140	0.220
120–180	0.025	0.040	0.063	0.100	0.160	0.250
180–250	0.029	0.046	0.072	0.115	0.185	0.290
250–315	0.032	0.052	0.081	0.130	0.210	0.320
315–400	0.036	0.057	0.089	0.140	0.230	0.360

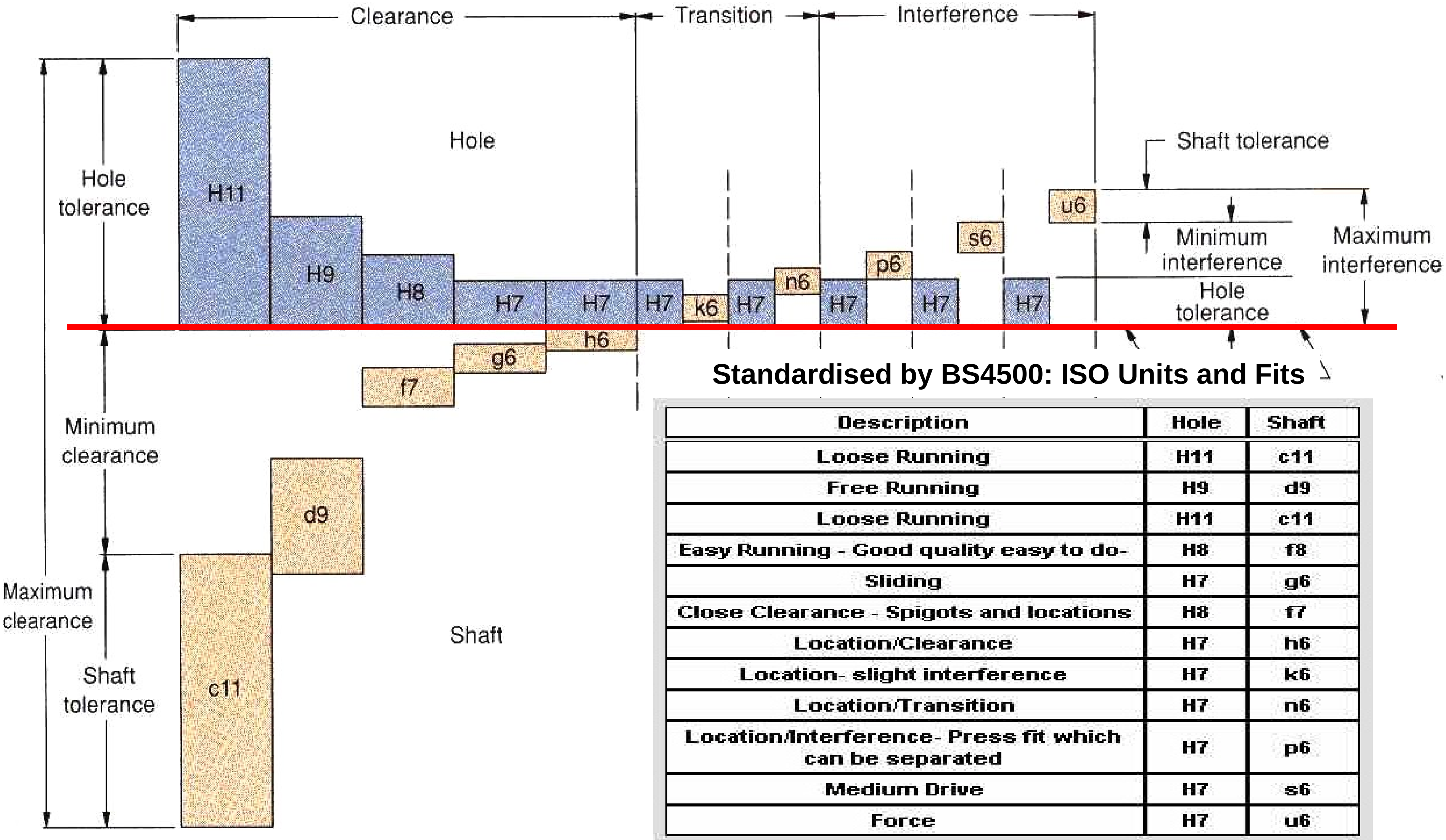
Preferred fits in the Basic-Hole System

TYPE OF FIT	DESCRIPTION	SYMBOL
Clearance	*Loose running* fit: for wide commercial tolerances or allowances on external members	H11/c11
	Free running fit: not for use where accuracy is essential, but good for large temperature variations, high running speeds, or heavy journal pressures	H9/d9
	Close running fit: for running on accurate machines and for accurate location at moderate speeds and journal pressures	H8/f7
	Sliding fit: where parts are not intended to run freely, but must move and turn freely and locate accurately	H7/g6
	Locational clearance fit: provides snug fit for location of stationary parts, but can be freely assembled and disassembled	H7/h6
Transition	*Locational transition* fit for accurate location, a compromise between clearance and interference	H7/k6
	Locational transition fit for more accurate location where greater interference is permissible	H7/n6
Interference	*Locational interference* fit: for parts requiring rigidity and alignment with prime accuracy of location but without special bore pressure requirements	H7/p6
	Medium drive fit: for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron	H7/s6
	Force fit: suitable for parts which can be highly stressed or for shrink fits where the heavy pressing forces required are impractical	H7/u6

To differentiate between holes and shafts, upper and lower case letters are used
H – Holes; h - Shafts



Preferred Hole Basis System of Fits



Standardised by BS4500: ISO Units and Fits

Description	Hole	Shaft
Loose Running	H11	c11
Free Running	H9	d9
Loose Running	H11	c11
Easy Running - Good quality easy to do-	H8	f8
Sliding	H7	g6
Close Clearance - Spigots and locations	H8	f7
Location/Clearance	H7	h6
Location- slight interference	H7	k6
Location/Transition	H7	n6
Location/Interference- Press fit which can be separated	H7	p6
Medium Drive	H7	s6
Force	H7	u6

Fundamental deviations for shafts

BASIC SIZES	UPPER-DEVIATION LETTER					LOWER-DEVIATION LETTER				
	c	d	f	g	h	k	n	p	s	u
0–3	−0.060	−0.020	−0.006	−0.002	0	0	+0.004	+0.006	+0.014	+0.018
3–6	−0.070	−0.030	−0.010	−0.004	0	+0.001	+0.008	+0.012	+0.019	+0.023
6–10	−0.080	−0.040	−0.013	−0.005	0	+0.001	+0.010	+0.015	+0.023	+0.028
10–14	−0.095	−0.050	−0.016	−0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
14–18	−0.095	−0.050	−0.016	−0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
18–24	−0.110	−0.065	−0.020	−0.007	0	+0.002	+0.015	+0.022	+0.035	+0.041
24–30	−0.110	−0.065	−0.020	−0.007	0	+0.002	+0.015	+0.022	+0.035	+0.048
30–40	−0.120	−0.080	−0.025	−0.009	0	+0.002	+0.017	+0.026	+0.043	+0.060
40–50	−0.130	−0.080	−0.025	−0.009	0	+0.002	+0.017	+0.026	+0.043	+0.070
50–65	−0.140	−0.100	−0.030	−0.010	0	+0.002	+0.020	+0.032	+0.053	+0.087
65–80	−0.150	−0.100	−0.030	−0.010	0	+0.002	+0.020	+0.032	+0.059	+0.102
80–100	−0.170	−0.120	−0.036	−0.012	0	+0.003	+0.023	+0.037	+0.071	+0.124
100–120	−0.180	−0.120	−0.036	−0.012	0	+0.003	+0.023	+0.037	+0.079	+0.144
120–140	−0.200	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.092	+0.170
140–160	−0.210	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.100	+0.190
160–180	−0.230	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.108	+0.210
180–200	−0.240	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.122	+0.236
200–225	−0.260	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.130	+0.258
225–250	−0.280	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.140	+0.284
250–280	−0.300	−0.190	−0.056	−0.017	0	+0.004	+0.034	+0.056	+0.158	+0.315
280–315	−0.330	−0.190	−0.056	−0.017	0	+0.004	+0.034	+0.056	+0.170	+0.350
315–355	−0.360	−0.210	−0.062	−0.018	0	+0.004	+0.037	+0.062	+0.190	+0.390
355–400	−0.400	−0.210	−0.062	−0.018	0	+0.004	+0.037	+0.062	+0.208	+0.435

Selected fits – Hole basis

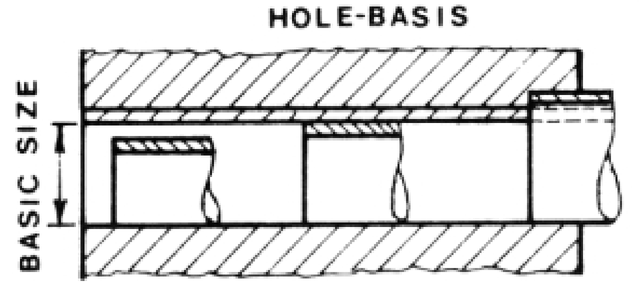
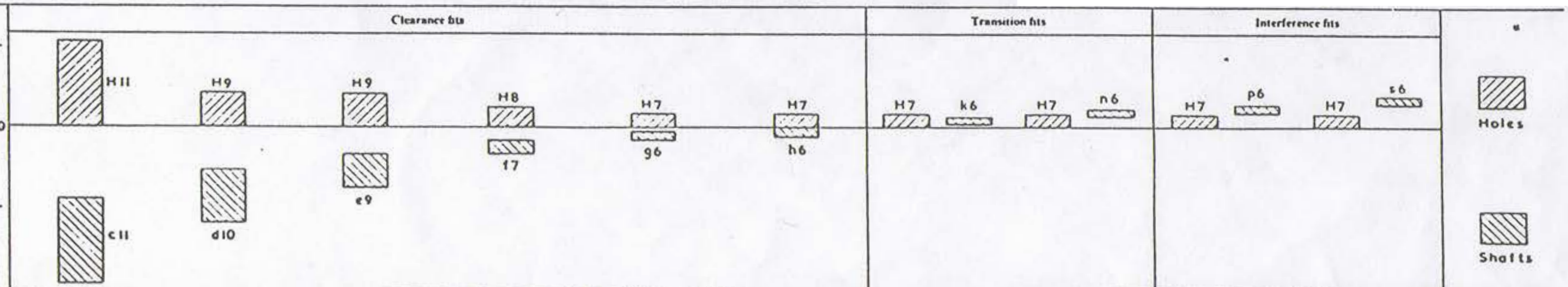


Diagram to scale for 25 mm. diameter



Nominal sizes		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Tolerance		Nominal sizes	
Over	To	H11	c11	H9	d10	H9	e9	H8	f7	H7	g6	H7	h6	H7	k6	H7	n6	H7	p6	H7	s6	Over	To
mm	mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	0 001 mm	mm	mm
—	3	+60 0	−60 −120	+25 0	−20 −60	+25 0	−14 −39	+14 0	−6 −16	+10 0	−2 −8	+10 0	−6 0	−10 0	+6 +0	+10 0	+10 +4	+10 0	+12 +6	+10 0	+20 +14	—	3
3	6	+75 0	−70 −145	+30 0	−30 −78	+30 0	−20 −50	+18 0	−10 −22	+12 0	−4 −12	+12 0	−8 0	+12 0	+9 +1	+12 0	+16 +8	+12 0	+20 +12	+12 0	+27 +19	3	6
6	10	+90 0	−80 −170	+36 0	−40 −98	+36 0	−25 −61	+22 0	−13 −28	+15 0	−5 −14	+15 0	−9 0	+15 0	+10 +1	+15 0	+19 +10	+15 0	+24 +15	+15 0	+32 +23	6	10
10	18	+110 0	−95 −205	+43 0	−50 −120	+43 0	−32 −75	+27 0	−16 −34	+18 0	−6 −17	+18 0	−11 0	+18 0	+12 +1	+18 0	+.23 +12	+18 0	+29 +18	+18 0	+39 +28	10	18
18	30	+130 0	−110 −240	+52 0	−65 −149	+52 0	−40 −92	+33 0	−20 −41	+21 0	−7 −20	+21 0	−13 0	+21 0	+15 +2	+21 0	+28 +15	+21 0	+35 +22	−21 0	+48 +35	18	30
30	40	+160 0	−120 −280	+62 0	−80 −180	+62 0	−50 −112	+39 0	−25 −50	+25 0	−9 −25	+25 0	−16 0	+25 0	+18 +2	+25 0	+33 +17	+25 0	+42 +26	−25 0	+59 +43	30	40
40	50	+160 0	−130 −290																			40	50
50	65	+190 0	−140 −330	+74 0	−100 −220	+74 0	−60 −134	+46 0	−30 −60	+30 0	−10 −29	+30 0	−19 0	+30 0	+21 +2	+30 0	+39 +20	+30 0	+51 +32	+30 0	+72 +53	50	65
65	80	+190 0	−150 −340																	+30 0	+78 +59	65	80
80	100	+220 0	−170 −390	+87 0	−120 −260	+87 0	−72 −159	+54 0	−36 −71	+35 0	−12 −34	+35 0	−22 0	+35 0	+25 +3	+35 0	+45 +23	+35 0	+59 +37	+35 0	+93 +71	80	100
100	120	+220 0	−180 −400																	+35 0	+101 +79	100	120
120	140	+250 0	−200 −450	+100 0	−145 −305	+100 0	−84 −185	+63 0	−43 −83	+40 0	−14 −39	+40 0	−25 0	+40 0	+28 +3	+40 0	+52 +27	+40 0	+68 +43	+40 0	+117 +92	120	140
140	160	+250 0	−210 −460																	+40 0	+125 +100	140	160
160	180	+250 0	−230 −480																	+40 0	+133 +108	160	180
180	200	+290 0	−240 −530	+115 0	170 −355	+115 0	100 −215	+72 0	−50 −96	+46 0	−15 −44	+46 0	−29 0	+46 0	+33 +4	+46 0	+60 +31	+46 0	+79 +50	+46 0	+151 +122	180	200
200	225	+290 0	260 550																	+46 0	+159 +130	200	225
225	250	+290 0	−280 −570																	+46 0	+169 +140	225	250
250	280	+320 0	−300 −620	+130 0	190 −400	+130 0	−110 −240	+81 0	−56 −108	+52 0	−17 −49	+52 0	−32 0	+52 0	+36 +4	+52 0	+66 +34	+52 0	+88 +56	+52 0	+190 +158	250	280
280	315	+320 0	−330 −650																	+52 0	+202 +170	280	315
315	355	+360 0	−360 720	+140 0	−210 −440	+140 0	−125 −265	+89 0	−62 −119	+57 0	−18 −54	+57 0	−36 0	+57 0	+40 +4	+57 0	+73 +37	+57 0	+98 +62	+57 0	+226 +190	315	355
355	400	+360 0	−400 −760																	+57 0	+244 +208	355	400
400	450	+400 0	−440 −840	+155 0	−230 −480	+155 0	−135 −290	+97 0	68 −131	+63 0	−20 −60	+63 0	−40 0	+63 0	+45 +5	+63 0	+80 +40	−63 0	+108 +68	+63 0	+272 +232	400	450
450	500	+400 0	−480 −880																	+63 0	+292 +252	450	500

Force Fit - example

Design web

Determine the “force fit” for a shaft and bearing hole that have basic diameter 32 mm and pressure fit H7/s6

	Hole	**Shaft**
Tolerance Grade	0.025 mm	0.016 mm
Upper deviation	0.025 mm	0.059 mm
Lower deviation	0.000 mm	0.043 mm
Max Diameter	32.025 mm	32.059 mm
Min Diameter	32.000 mm	32.043 mm
Average Diameter	32.013 mm	32.051 mm

Max Clearance $C_{max} = D_{max} - d_{min} = 0.051$ mm

Min Clearance $C_{min} = D_{min} - d_{max} = 0.030$ mm

…e	Tolerance		Nominal sizes	
p6	H7	s6	Over	To
…001 mm	0 001 mm	0 001 mm	mm	mm
+ 12 + 6	+ 10 0	+ 20 + 14	—	3
+ 20 + 12	+ 12 0	+ 27 + 19	3	6
+ 24 + 15	+ 15 0	+ 32 + 23	6	10
+ 29 + 18	+ 18 0	+ 39 + 28	10	18
+ 35 + 22	− 21 0	+ 48 + 35	18	30
+ 42 + 26	− 25 0	+ 59 + 43	30	40
			40	50
+ 51 + 32	+ 30 0	+ 72 + 53	50	65
	+ 30 0	+ 78 + 59	65	80
+ 59 + 37	+ 35 0	+ 93 + 71	80	100
	+ 35 0	+ 101 + 79	100	120

Hole	**Shaft**
$32^{+0.025}_{+0.000}$	$32^{+0.059}_{+0.043}$

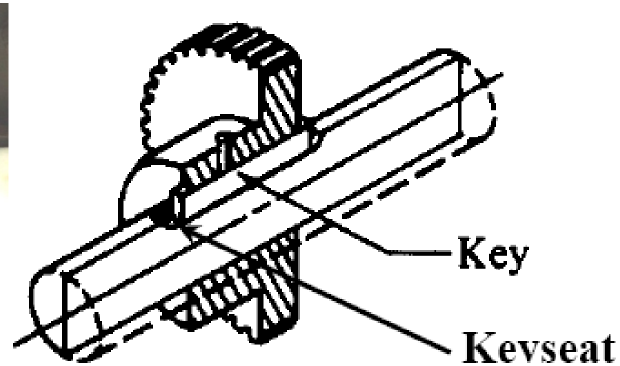
Design web

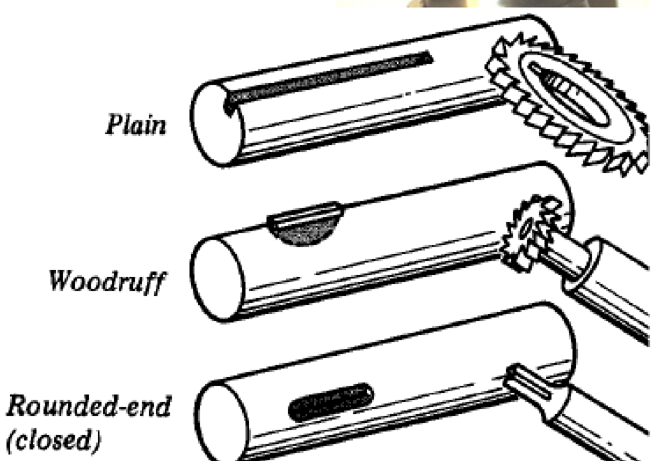
Keys and pins

Used on shafts to secure rotating elements; gears, pulleys, wheels.

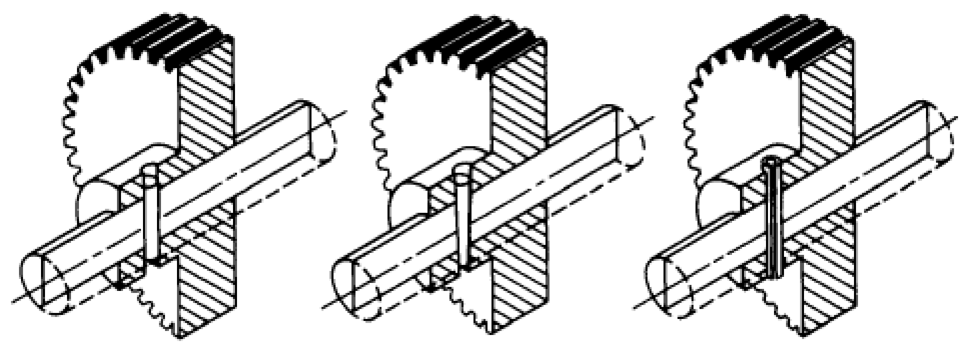
Keys – transmit torque between the shaft and the rotating element

Pins – axial positioning, transfer of torque or/and thrust.



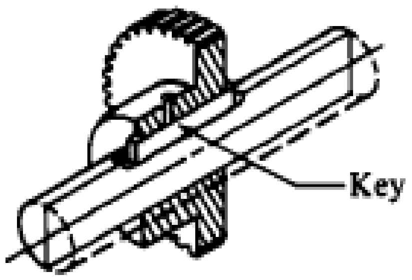


Shear Pins

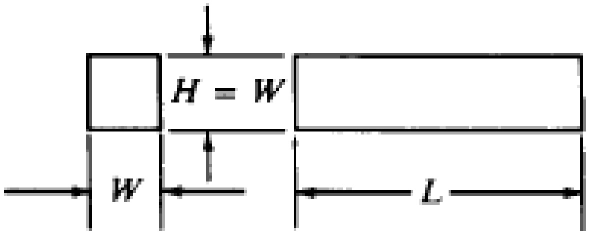


(a) Cylindrical pin (b) Taper pin (c) Spring pin

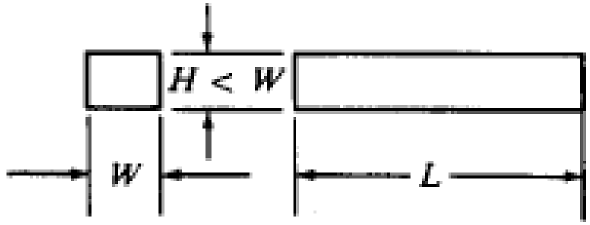
Strength of a key



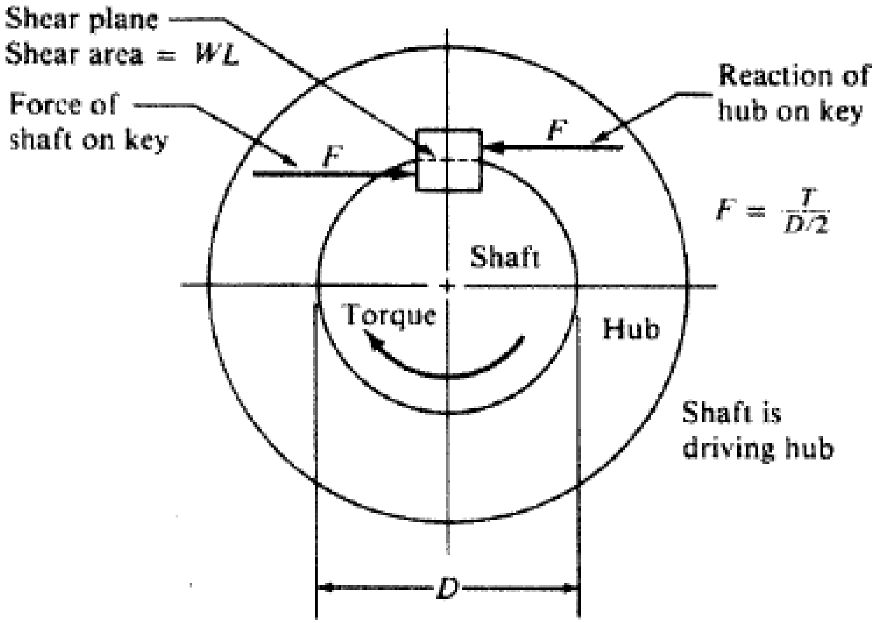
(a) Key and keyseat applied to a gear and shaft



(b) Square key



(c) Rectangular key



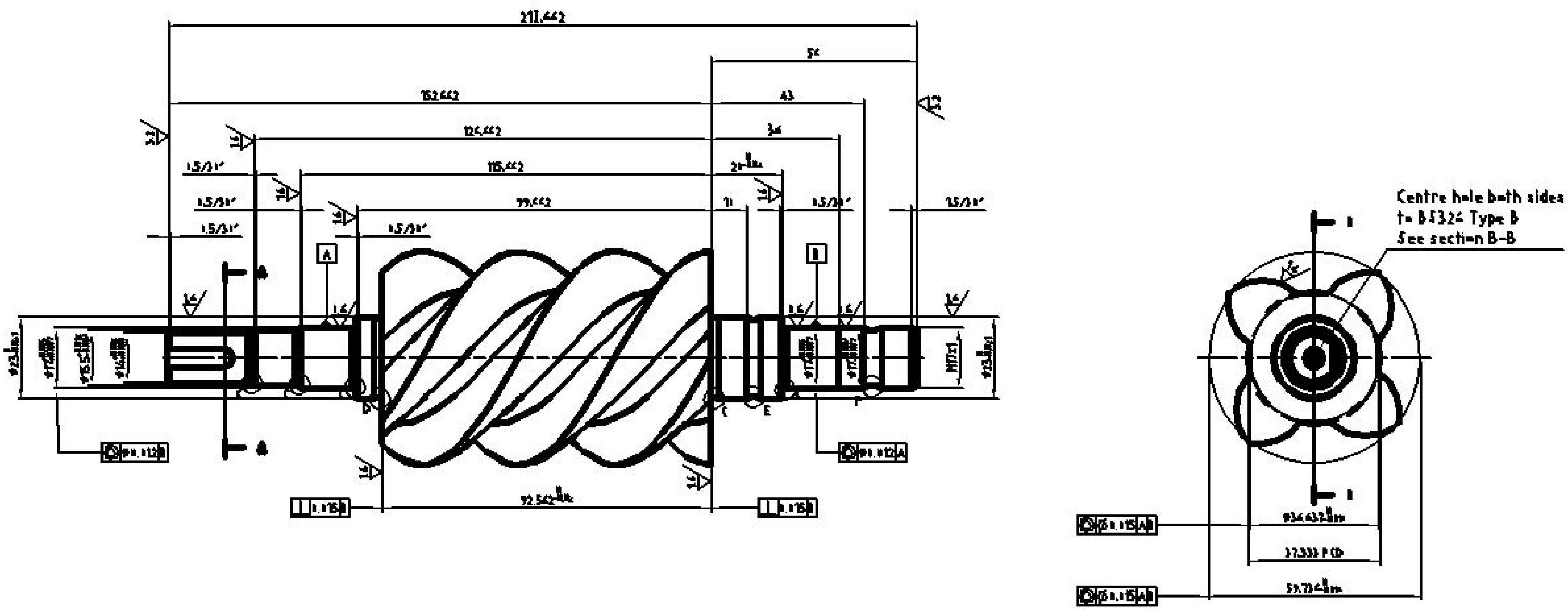
$$W \approx D/4$$

Allowed torque on the key:

$$T = F\frac{D}{2}; \qquad A = WL$$

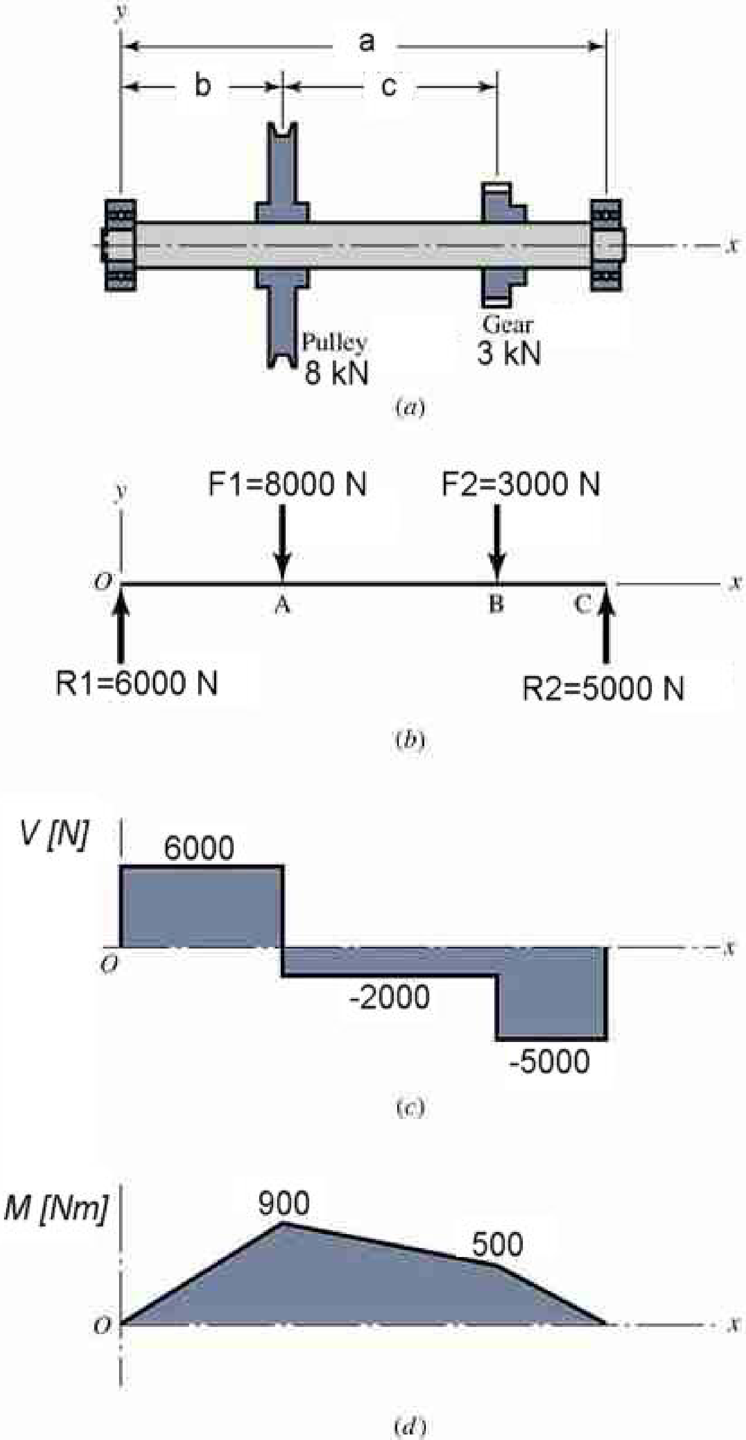
$$\tau = \frac{F}{A} = \frac{2T}{DWL} \Rightarrow T = \frac{DWL\,S_y}{4 f_s}$$

Drawing and dimensioning



Design web

Example



Determine the diameter for the solid round shaft 450 mm long, as shown in Figure. The shaft is supported by self-aligning bearings at the ends. Mounted upon the shaft are a V-belt pulley, which contributes a radial load of F_1=8kN to the shaft, and a gear which contributes a radial load of F2=3kN. The two loads are in the same plane and have the same directions. The allowable bending stress (strength) is S=70 MPa.

F1=8 kN

F2=3 kN

a=450 mm

b=150 mm

c=200 mm

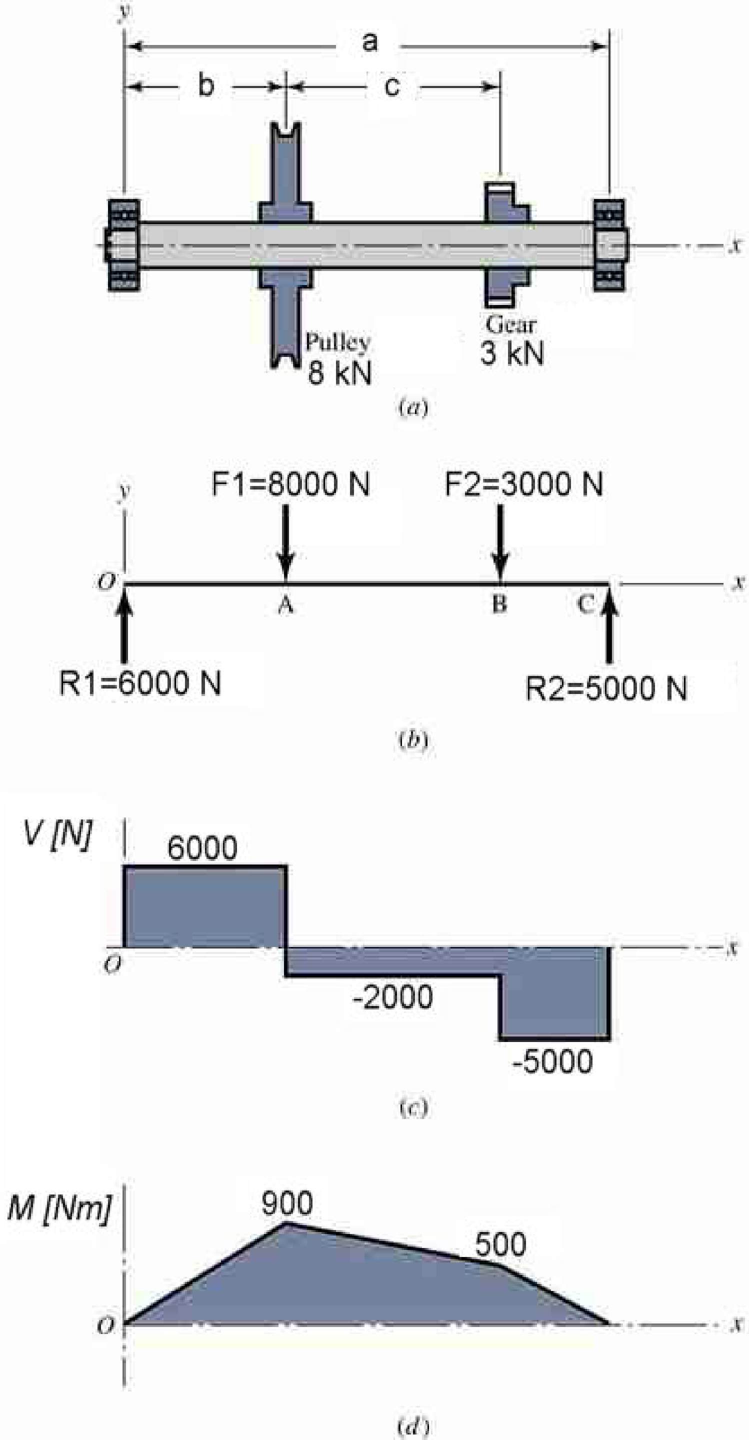
S=70 MPa

d=?

SOLUTION:

Assumptions

- the weight of the shaft is neglected
- the shaft is designed for the normal bending stress in the location of max. bending moment



Design web

Solution

$$\sum M_c = -a\,R_1 + (a-b)\,F_1 + (a-b-c)\,F_2$$

$$R_1 = 6\,kN$$

$$R_2 = 5\,kN$$

$$M_{\max} = M_A = b \cdot R_1 = 900\,Nm$$

Second moment of area (moment of inertia)

Section modulus =--

max span

$$I = \frac{\pi d^4}{64} \qquad c = \frac{d}{2}$$

$$Z = \frac{I}{c} = \frac{\pi d^3}{32} \approx 0.1 d^3$$

Stress = Strain = Bending moment / section modulus

$$S = \sigma = \frac{Mc}{I} = \frac{M}{Z} = \frac{900}{0.1 \cdot d^3} = 70 \cdot 10^6$$

$$d = \sqrt[3]{\frac{900}{70 \cdot 10^5}} = 0.050\,m = 50mm$$