Mechanical Analysis and Design

ME 2104

Lecture 4

Mechanical Analysis
Gear trains

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www.staff.city.ac.uk/~ra600/intro.htm

Plan for the analysis of mechanical elements

Objective:

Procedures for design and selection of mechanical elements

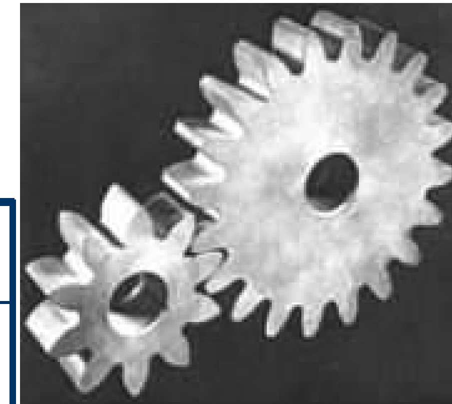
- Week 1 – Shafts and keyways
- Week 2 – Bearings and screws
- Week 3 – Belt and chain drives
- Week 4 – Gears and gear trains
- Week 5 – Design Project Review

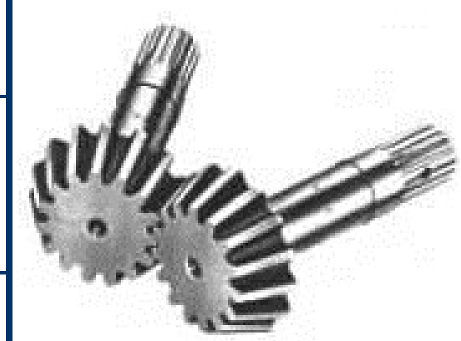
Plan for this week

- Gears
- Gear trains
- Examples

Gear types

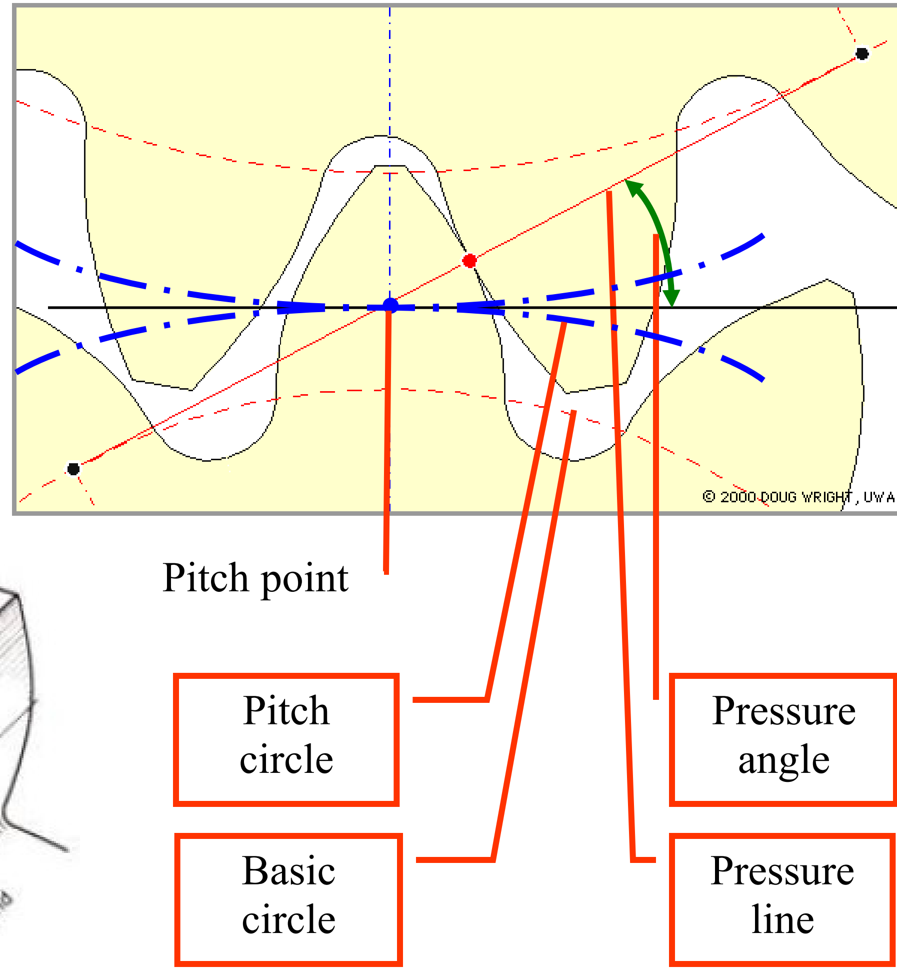
Gear	Input/Output		Motion Axis	Loads
Spur	Rotary	Rotary	Parallel	Tangent
Bevel	Rotary	Rotary	Angled	Tangent
Helical	Rotary	Rotary	Parallel or Crossed	Tangent and Axial
Rack	Rotary	Linear	90°	Tangent
Worm	Rotary	Rotary or Linear	90°	Tangent Not back drivable

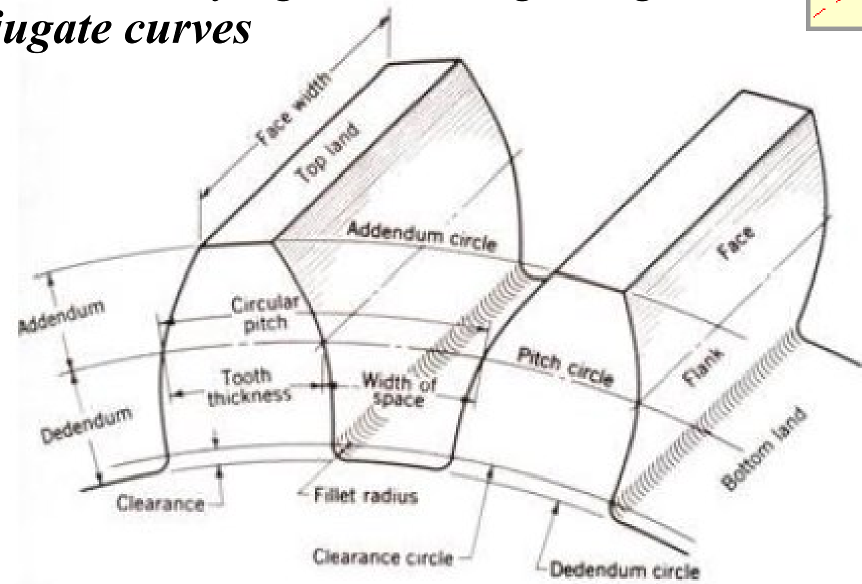




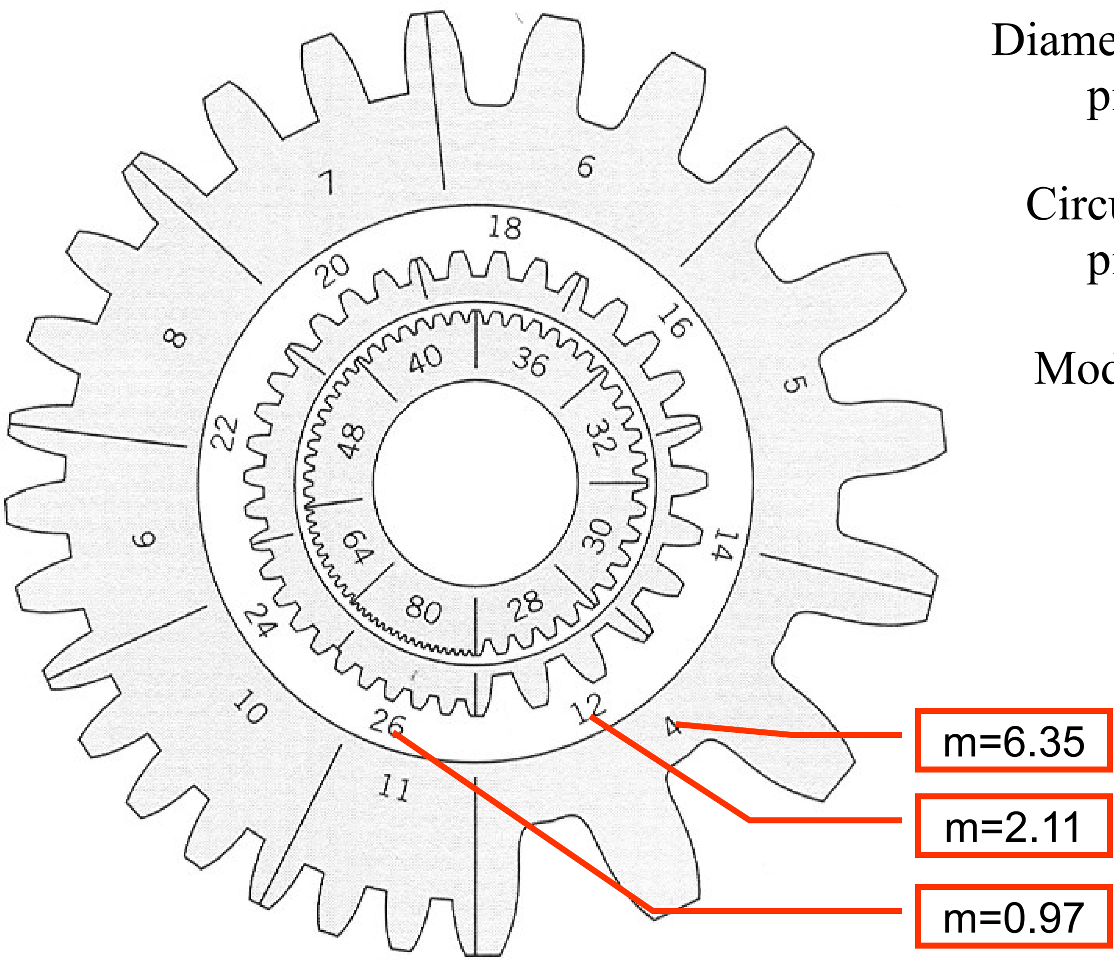
How gears work

- Law of Gearing:
  - *A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centres called the **pitch point.***
  - Any two curves or profiles engaging each other and satisfying the law of gearing are ***conjugate curves***





Module and Pitch



Diametral pitch $P = \frac{N_G}{D_G} = \frac{N_P}{D_P} \left[in^{-1} \right]$

Circular pitch $p = \frac{\pi}{P} = \pi\, m = \pi \frac{D}{N} [mm]$

Module $m = \frac{D}{N} \quad [mm];\ m = \frac{25.4}{P}$

Standard modules are 0.5, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6

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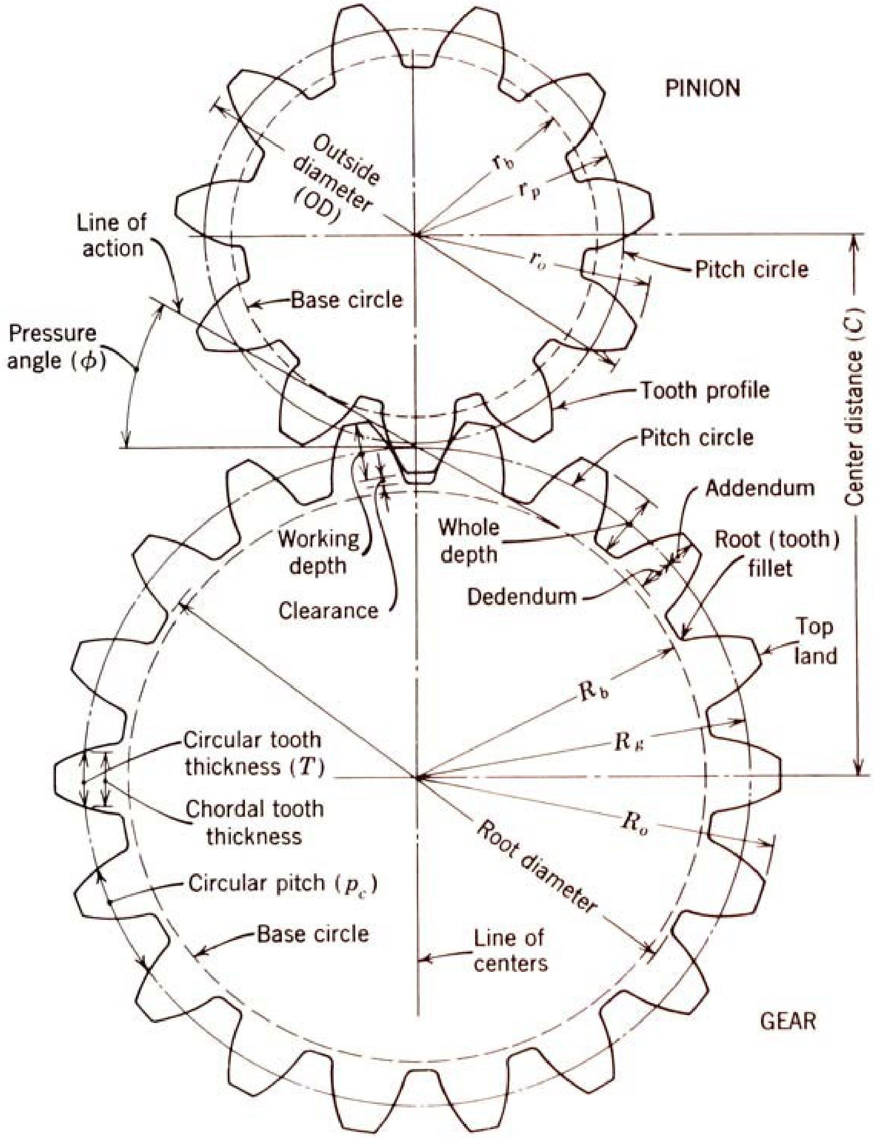
Relations between gear parameters



Centre distance: $C = \dfrac{D_P + D_G}{2}$

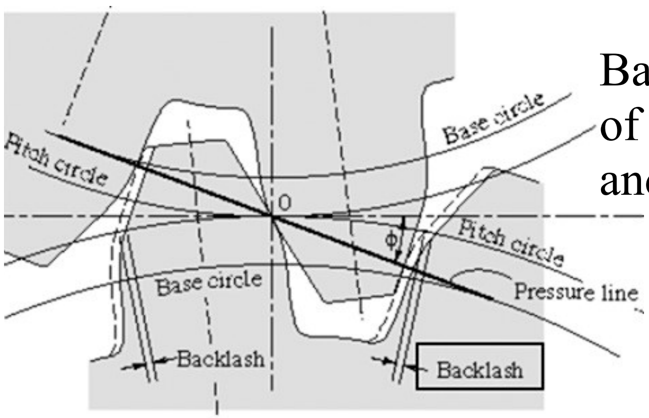
Pressure angle: 20° (14.5°)

Addendum: a = module

Dedendum: b = 1.25 module

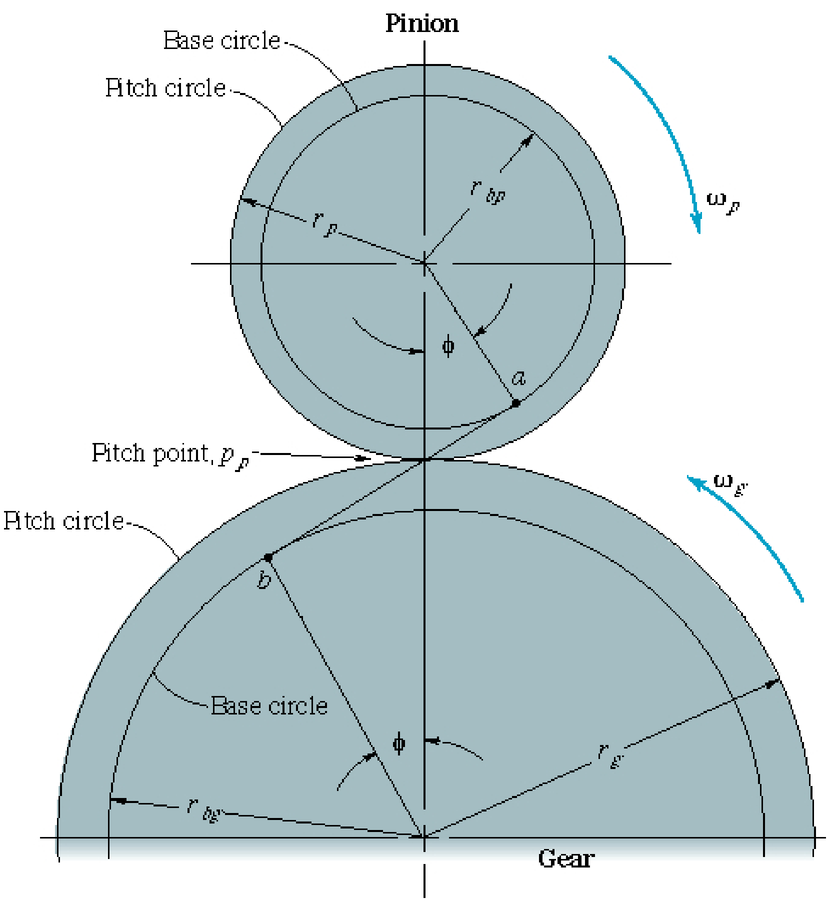
Clearance: c = 0.25 module

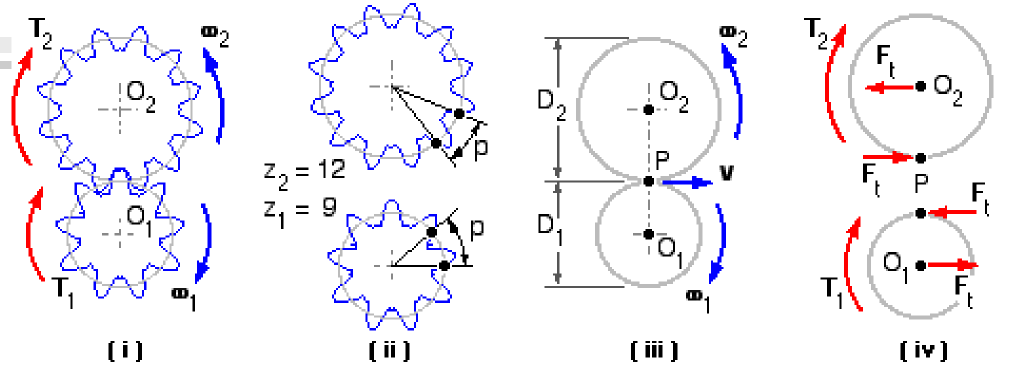
Backlash: clearance measured on the pitch circle of a driving gear



Backlash is function of module and centre distance

Relations between gear parameters





$$P_{ow} = \omega T = \omega_G T_G = \omega_P T_P$$

$$v = \omega_G \frac{D_G}{2} = \omega_P \frac{D_P}{2}$$

$$\omega_G = n_G \frac{\pi}{30};\ \omega_P = n_P \frac{\pi}{30}$$

$$GR = \frac{D_G}{D_P} = \frac{N_G}{N_P} = \frac{n_P}{n_G} = \frac{T_G}{T_P}$$

Gear ratio

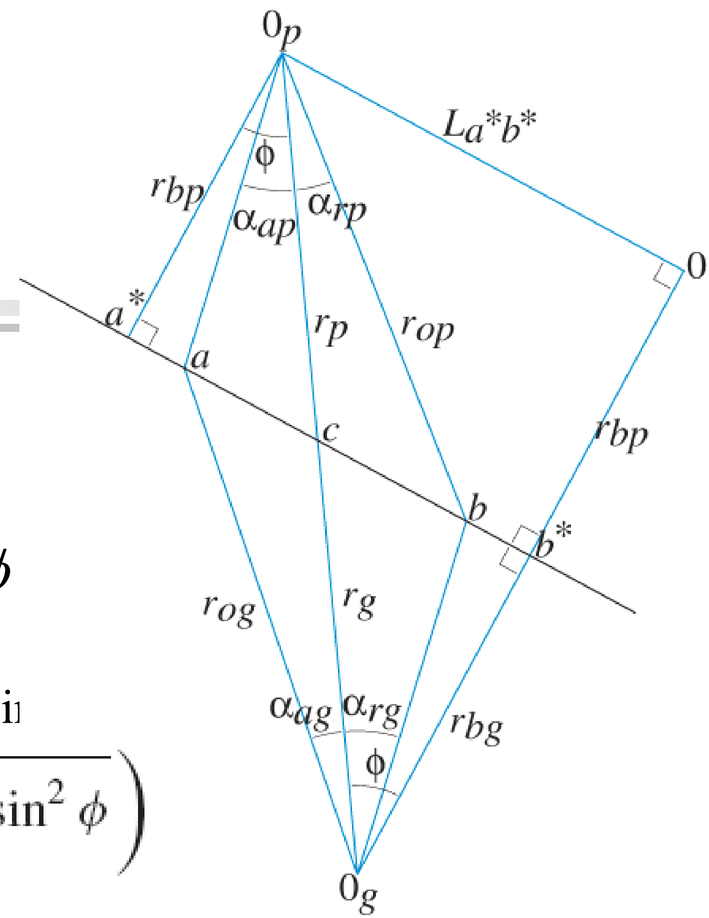
$$r_b = r\cos\phi$$

Radius of the base circle

$$p_b = p\cos\phi = \pi\frac{D}{N}\cos\phi$$

Pitch of the base circle

Contact ratio and interference



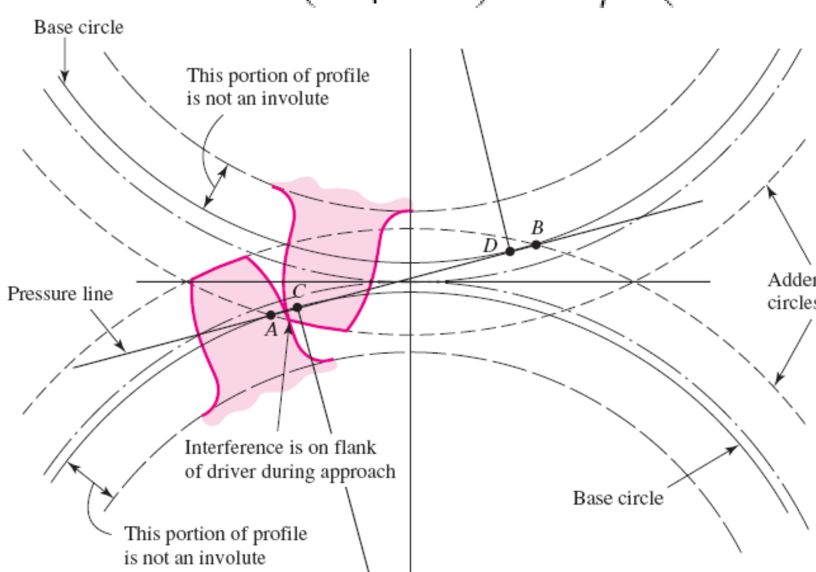
Contact ratio:

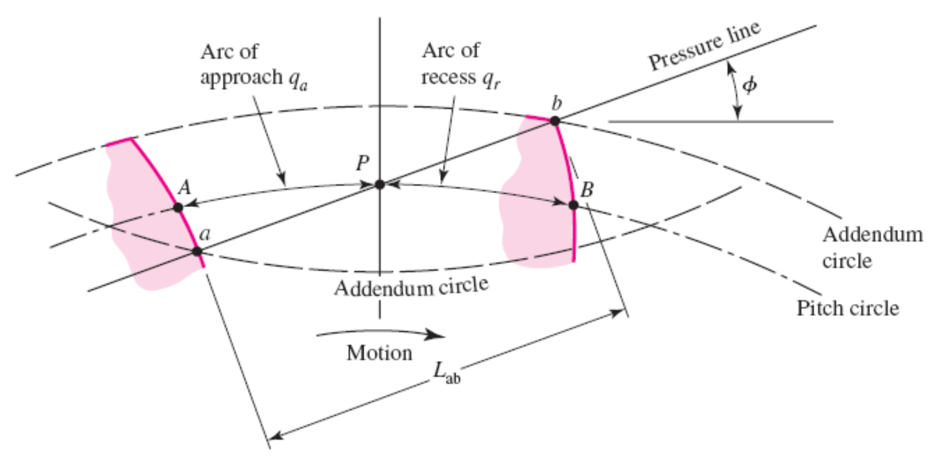
$$c_r = \frac{L_{ab}}{p_b} = \frac{L_{ab}}{p \cos\phi}$$

$$L_{ab} = \sqrt{r_{op}^2 + r_{bp}^2} + \sqrt{r_{og}^2 + r_{bg}^2} - C_d \sin\phi$$

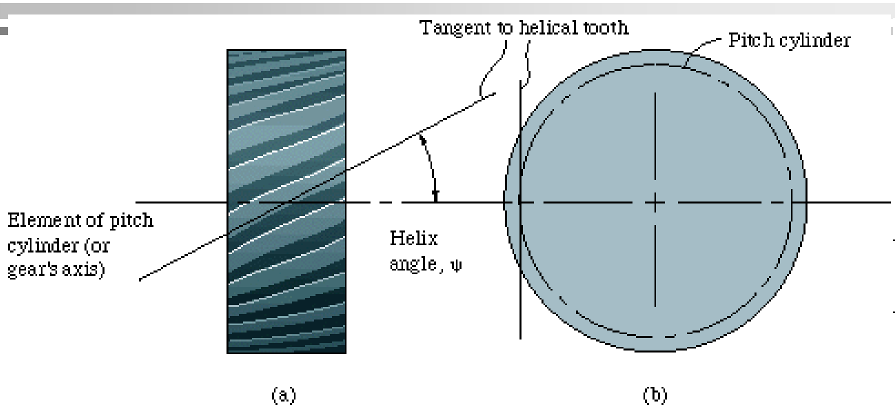
Interference (the smallest number of teeth for contact without i

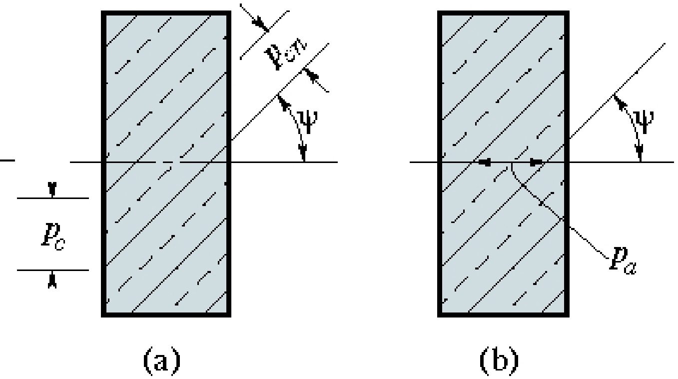
$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$





Helical gears





Normal circular pitch $p_{cn} = p_c \cos\psi$

Normal diametral pitch $p_{dn} = p_d \cos\psi$

Axial Pitch $p_a = p_c \cot\psi = \dfrac{p_{cn}}{\sin\psi}$

Axial load $F_a = F_t \tan\psi$

Radial load $F_r = F_t \tan\phi$

Normal load $F = \dfrac{F_t}{\cos\phi\cos\psi}$

$N_n = \dfrac{N}{\cos^3\phi}$ Number of teeth for helical gear

$\tan\phi = \dfrac{\tan\phi_n}{\cos\psi}$ Normal pressure angle

$c_d = \dfrac{D_p + D_g}{2\cos\psi} = \dfrac{N_p + N_g}{2\cos\psi}$ b_w – gear width

$C_{rt} = C_r + C_{ra}$ Contact ratio for helical gear

$C_{ra} = \dfrac{b_w \tan\psi}{p_c}$ Axial contact ratio

Example 11 –helical gear

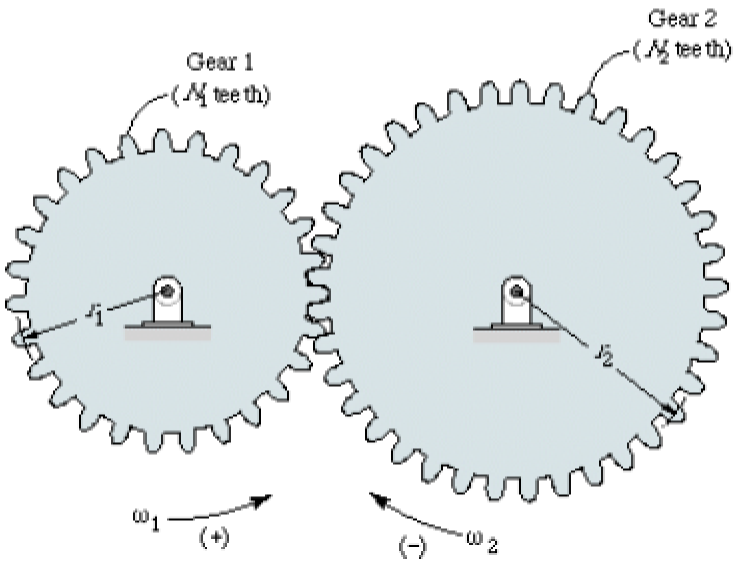
An involute gear drives a high-speed centrifuge. The speed of the centrifuge is 18000 rpm. It is driven by a 3000 rpm electric motor through 6:1 speedup gearbox. The pinion has 21 teeth and the gear has 126 teeth with a diametral pitch of 14 per inch. The width of gears is 45.72 mm and the pressure angle is 20°. Power of the electric motor is 10kW.

Determine :

a) A contract ratio of a spur gear
b) A contact ratio of a helical gear with helix angle of 30°.
c) A helix angle if the contact ratio is 3
d) Axial, radial and contact (normal) force for both helix angles.

a) $C_r=1.712$
b) $C_{r1}=6.343$
c) $\psi=9.122^o$
d) $F_{a1}=160.8\ N$
$F_{a2}=44.7\ N$

External Meshing



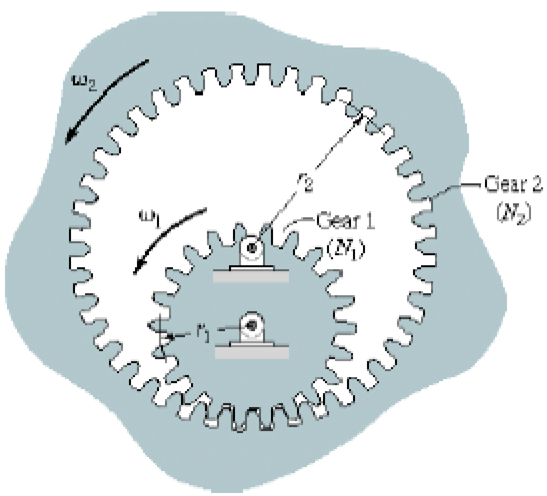
Gear ratio

$$Z_{21} = \frac{\omega_2}{\omega_1} = -\frac{N_2}{N_1}$$

Center distance

$$c_d = r_1 + r_2$$

Internal Meshing



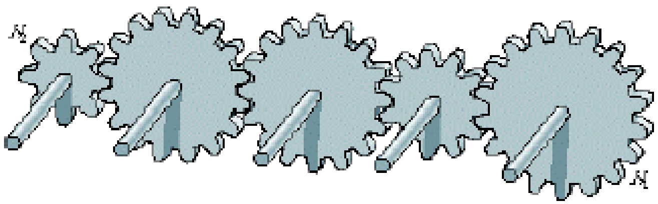
Gear ratio

$$Z_{21} = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1}$$

Center distance

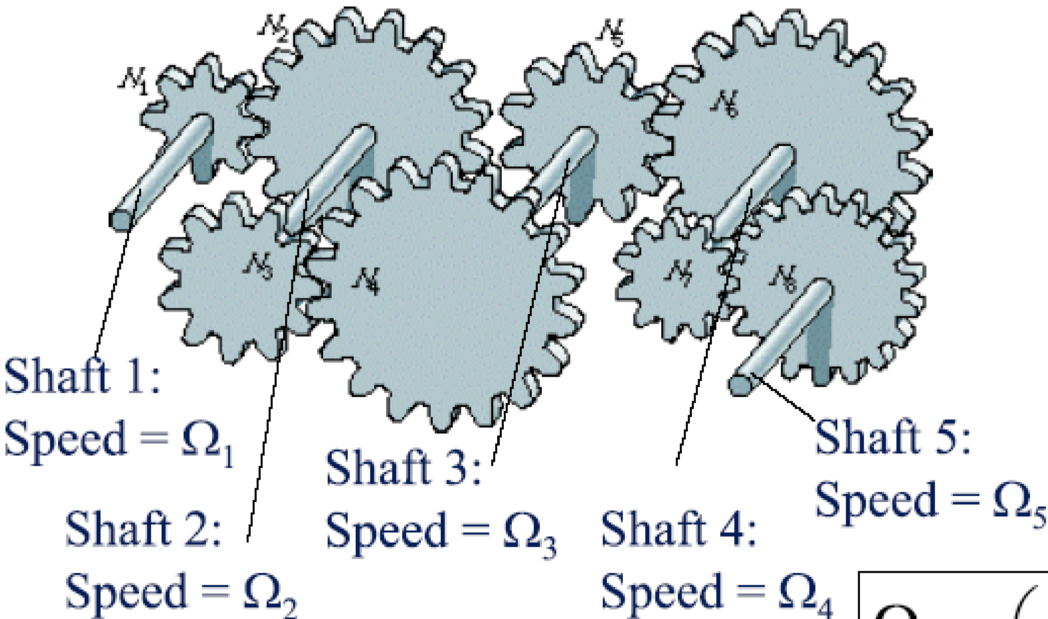
$$c_d = r_1 - r_2$$

Simple Gear Trains



Gear ratios $Z_{ji} = \frac{\omega_j}{\omega_i} = \frac{N_i}{N_j}$

Compound Gear Trains



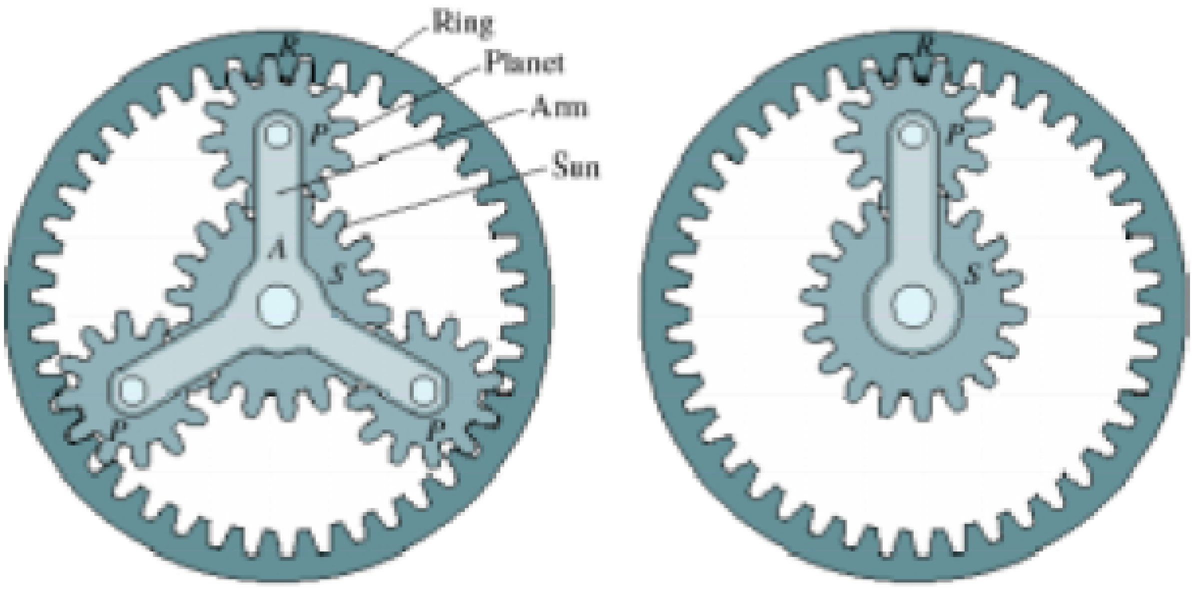
Gear ratio

$$Z_{51} = \frac{\Omega_5}{\Omega_1}$$

$$\frac{\Omega_5}{\Omega_1} = \frac{\Omega_5}{\Omega_4}\frac{\Omega_4}{\Omega_3}\frac{\Omega_3}{\Omega_2}\frac{\Omega_2}{\Omega_1}$$

$$\frac{\Omega_5}{\Omega_1} = \left(-\frac{N_7}{N_8}\right)\left(-\frac{N_5}{N_6}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_1}{N_2}\right)$$

Planetary Gear Train



(a) (b)

To relate the rpm of the ring to the rpm's of the arm and sun:

$$\frac{\omega_{ring} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} = -\frac{N_{sun}}{N_{ring}}$$

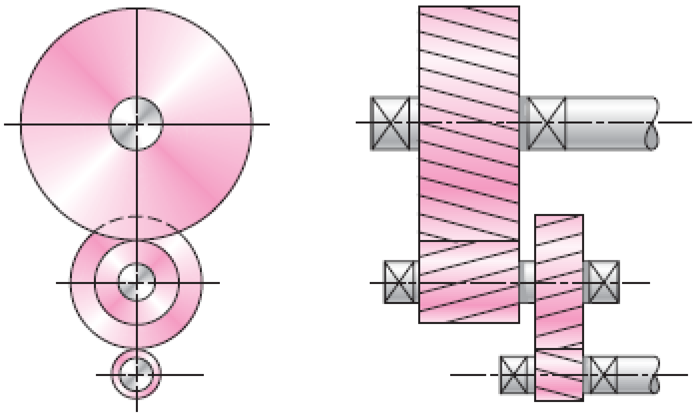
To relate the rpm of the planets to the rpm's of the arm and sun:

$$\frac{\omega_{planet} - \omega_{arm}}{\omega_{sun} - \omega_{arm}} = -\frac{N_{sun}}{N_{planet}}$$

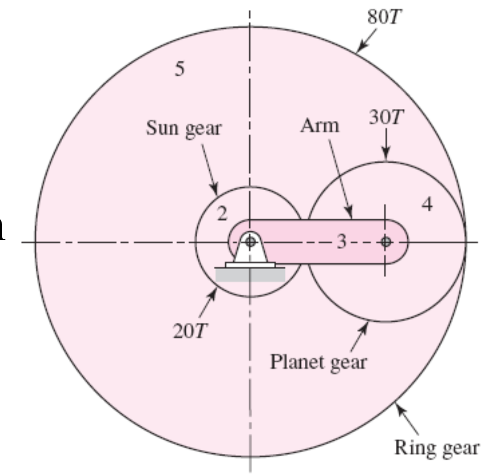
Relationship between the numbers of teeth on the ring, planets and sun:

$$N_{ring} = N_{sun} + 2N_{planet}$$

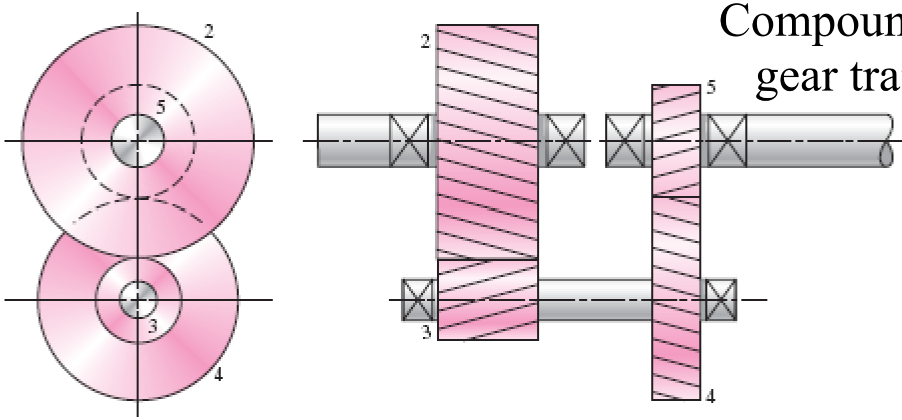
Examples of gear trains



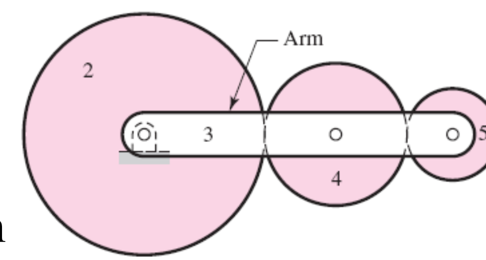
Two stage compound Gear train



Planetary gear train



Compound reverted gear train



Planetary gear train on the arm

Example 12 – Compound gear train

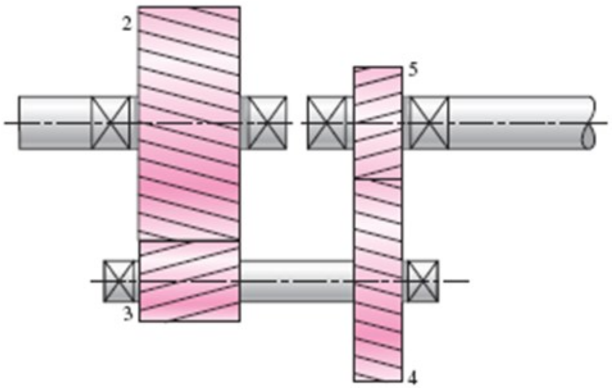
A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line.

Governing equations are:

$$N_2/N_3 = 6$$

$$N_4/N_5 = 5$$

$$N_2 + N_3 = N_4 + N_5$$



Specify appropriate teeth numbers.

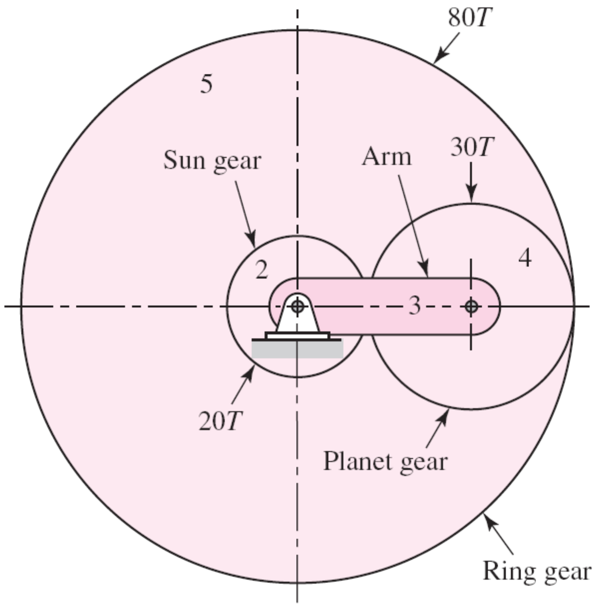
$N_2 = 108; N_3 = 18; N_4 = 105; N_5 = 21$

Example 13 – Planetary gear train

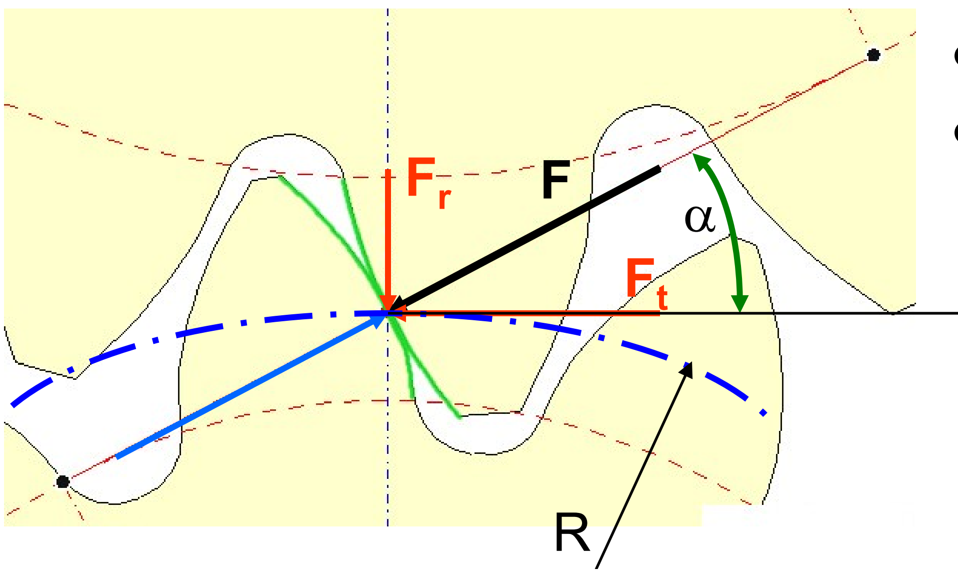
The sun gear in the figure is the input, and it is driven clockwise at 100 rpm. The ring gear is held stationary by being fastened to the frame.

Find the rev/min and direction of rotation of the arm and gear 4.

n_3 = - 20 rpm, n_4 = 33.3 rpm,



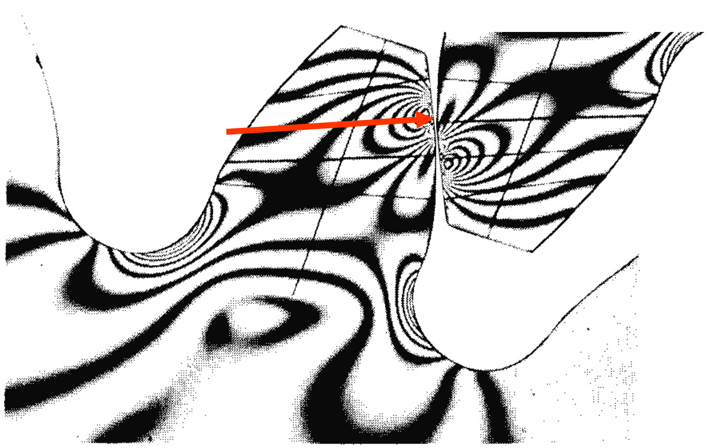
Gear Force Calculation



- Stress based on the *Force* acting
- The force is caused by the transmitted torque. That force always acts along the pressure line.

$$T = P/\omega = F_t R \rightarrow \quad F_t = \frac{30P}{\pi n R} [N]$$

$$F = F_t / \cos\alpha \rightarrow \quad F = \frac{30P}{\pi n R \cos\alpha}$$

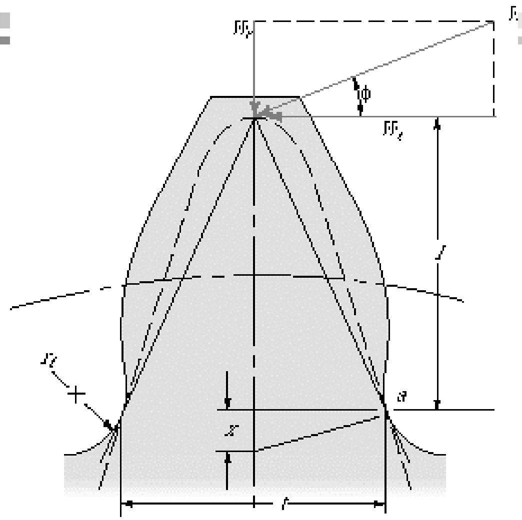


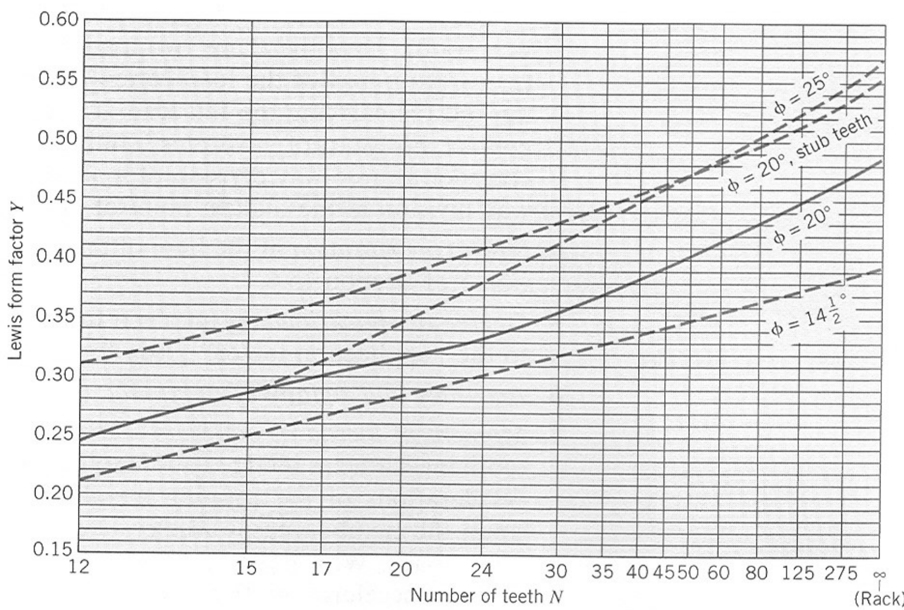
- The force induces stress concentration on gear.

We need to answer:

How much power a pair of gears in question can transfer?

Basic gear stress calculation





- Basic analysis of gear-tooth is based on Lewis Equation which has following assumptions:
  - » The full load is applied on the tip of a single tooth (the worse case)
  - » Radial component is negligible
  - » The load is distributed uniformly along the teeth width
  - » Friction forces are negligible
  - » Stress concentration is negligible.

Basic stress in teeth

$$\sigma_B = \frac{F_t}{mbY}$$

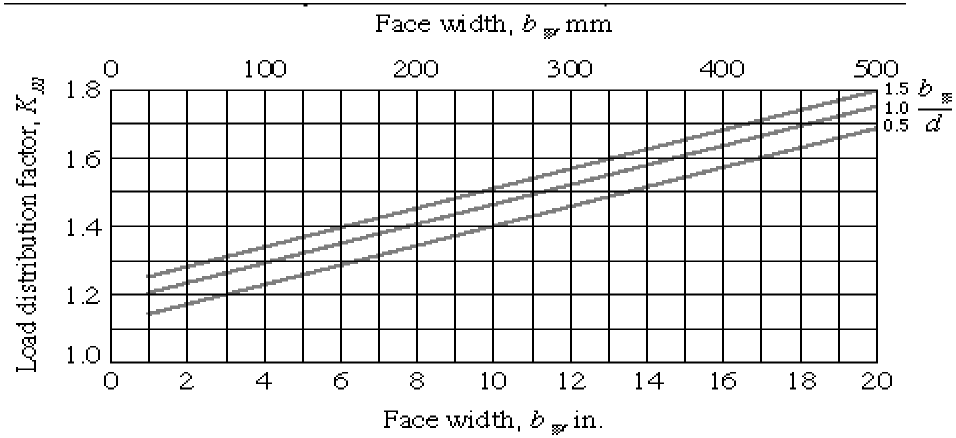
Power transmitted

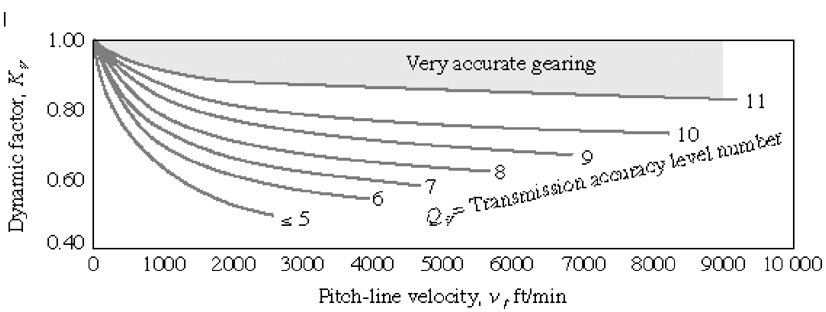
$$P_B = \frac{S_y}{f_s} n\, D\, m\, b\, Y$$

Correction factors

	Driven Machines			
Power Source	Uniform	Light shock	Moderate shock	Heavy shock
	Application factor, K_a			
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Diametral pitch p_d, in.$^{-1}$	Module, m, mm	Size factor, K_s
≥5	≤5	1.00
4	6	1.05
3	8	1.15
3	12	1.25
1.25	20	1.40





Basic stress must be corrected for:
- Shocks,
- Size effects,
- Uneven load distribution,
- Dynamic effects.

$$\sigma = \sigma_B \frac{K_a K_s K_m}{K_v}$$

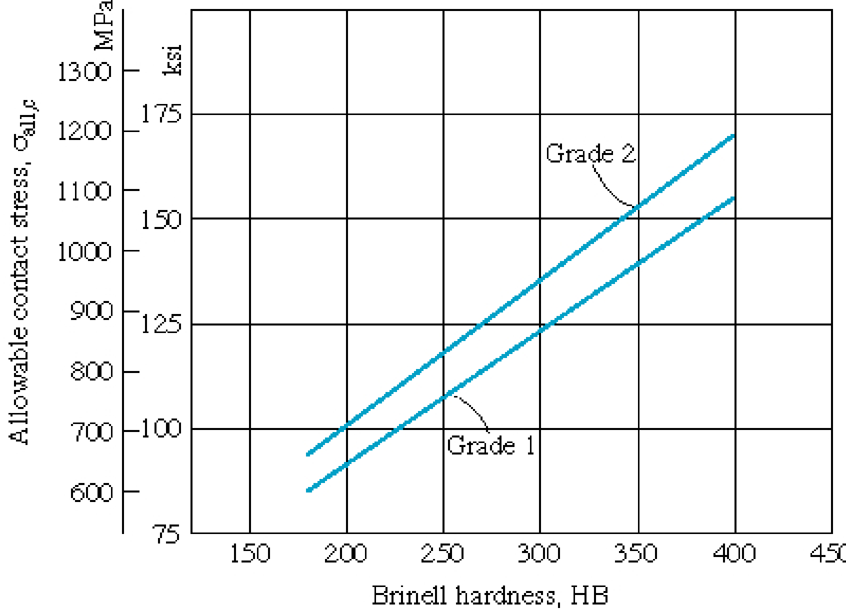
$$P = P_B \frac{K_a K_s K_m}{K_v}$$

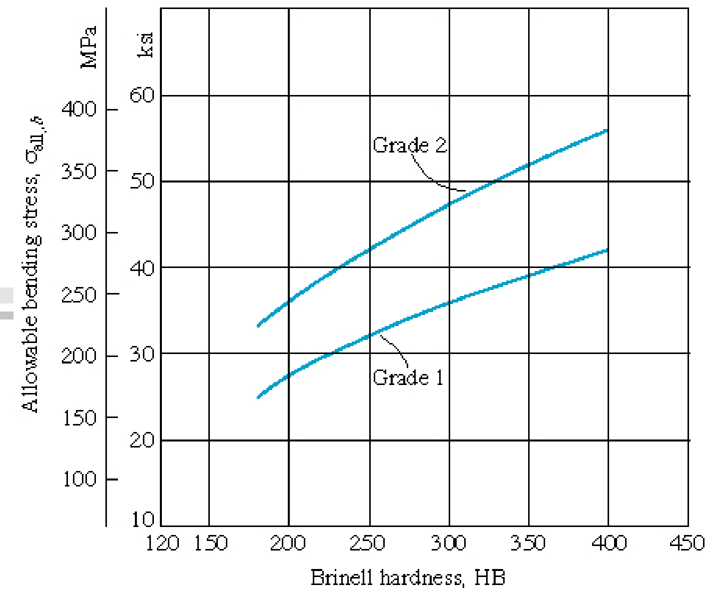
K_a – Application factor

K_s – Size factor

K_m – Load distribution factor

K_v - Dynamic factor





Average Mechanical Properties and Typical Uses of Ductile (Nodular) Iron

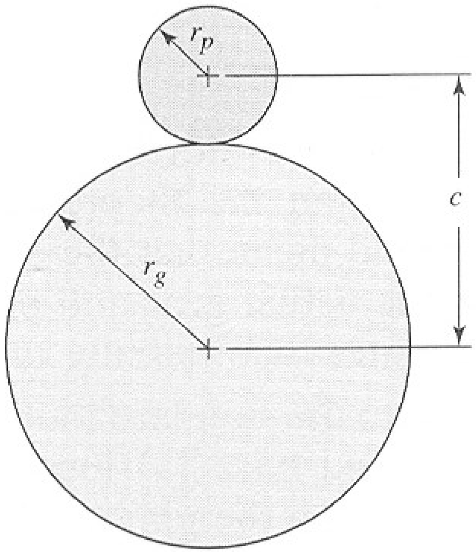
Grade[a]	Brinell Hardness, H_B	Elongation (%) (in 50 mm)	Poisson's Ratio	Tensile Modulus GPa	Tensile Modulus 10^6 psi	Typical Uses
60-40-18	167	15.0	0.29	169	24.5	Valves and fittings for steam and chemicals
65-45-12	167	15.0	0.29	168	24.4	Machine components subject to shock and fatigue
80-55-06	192	11.2	0.31	168	24.4	Crankshafts, gears, rollers
120-90-02	331	1.5	0.28	164	23.8	Pinions, gears, rollers, slides

Grade	Tensile Strength Ultimate MPa	Tensile Strength Ultimate 10^6 psi	Tensile Strength Yield MPa	Tensile Strength Yield 10^6 psi	Compressive Strength: Ultimate MPa	Compressive Strength: Ultimate 10^6 psi	Torsional Strength Ultimate MPa	Torsional Strength Ultimate 10^6 psi	Torsional Strength Yield MPa	Torsional Strength Yield 10^6 psi
60-40-18	461	66.9	329	47.7	359	52.0	472	68.5	195	28.3
65-45-12	464	67.3	332	48.2	362	52.5	475	68.9	297	30.0
80-55-06	559	81.8	362	52.5	386	56.0	504	73.1	193	28.0
120-90-02	974	141.3	864	125.3	920	133.5	875	126.9	492	71.3

[a]The first two sections of grade number indicate minimum values (in ksi) of tensile ultimate and yield strengths.

Source: ASM Metals Reference Book, American Society for Metals, Metals Park, OH, 1981.

Example 14 – Gear stress calculation



Pinion A and gear B are shown in figure. Pinion A rotates at 1750 rpm, driven directly by an electric motor. The driven machine is an industrial saw consuming 20 kW. The following conditions are given:

N_P=20	m=3 mm	Q_v=6
N_G=70	b=38 mm	f_s=1.5
n_p=1750 rpm	Pow=20 kW	

What is the centre distance? Compute the stress due to bending in the pinion and gear and find required Brinell hardness for this application.

SOLUTION:

Centre distance:

$$c = {(D_P + D_G)}/{2} = m{(N_P + N_G)}/{2} = 3{90}/{2} = 135[mm]$$

The pitch diameter of the pinion is:

$$D_P = mN_P = 3 \cdot 20 = 60[mm] = 0.06[m]$$

The pitch velocity is:

$$v_P = \frac{\pi n_P D_P}{60} = \frac{\pi \cdot 1750 \cdot 0.06}{60} = 5.5[m/s] = 1090[ft/min]$$

Transferred load (Tangential force):

$$F_t = \frac{60 Pow}{\pi n_P D_P} = \frac{60 \cdot 20000}{\pi \cdot 1750 \cdot 0.060} = 3638[N]$$

Example 14 – cont.

From the diagram: $Y_P=0.34$ and $Y_G=0.42$

Basic bending stress is: pinion –

$$\sigma_{BP}=\frac{F_t}{mbY_P}=\frac{3638}{0.003\cdot 0.038\cdot 0.34}=94\cdot 10^6\,[Pa]$$

gear -

$$\sigma_{BG}=\frac{F_t}{mbY_G}=\frac{3638}{0.003\cdot 0.038\cdot 0.42}=76\cdot 10^6\,[Pa]$$

Correction factors are:
(from diagrams and tables)

Application factor	$K_a=1.5$
Size factor	$K_s=1.0$
Load distribution	$K_m=1.2$
Dynamic factor	$K_v=0.68$

Corrected pinion bending stress:

$$\sigma_P=\sigma_{BP}\frac{K_aK_sK_m}{K_v}=2.64\cdot 94\cdot 10^6=248\,[MPa]$$

Allowable stress required for this application:

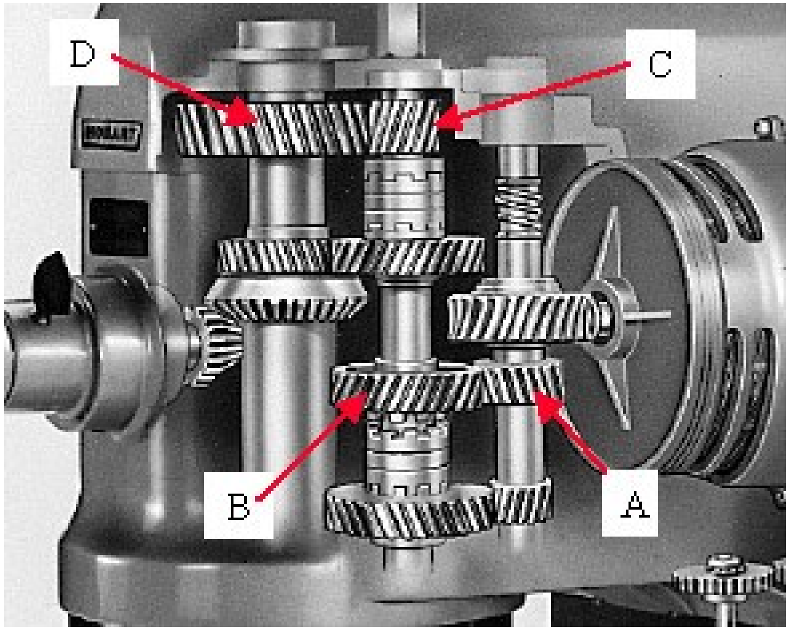
$$S=f_s\sigma_P=248\cdot 1.5=372\,[MPa]$$

From the diagram, any material with Brinel hardness higher than HB=400 will satisfy application requirements.

Example 15 – Industrial mixer

A gear train is made up of helical gears with their shafts in a single plane, such as the arrangement in the vertical mixer shown. The gears have a normal pressure angle of 20° and a 30° helix angle. The middle shaft is an idler. Gears AB and CD are engaged, the others in the illustration are not in contact. The module of all gears is 2 mm. The table gives the number of teeth per gear. Gear A exerts a load of 1200 N onto gear B. The shaft containing gear A is driven by the motor at a speed of 200 rpm. All gears are Grade 2 and have been hardened to HB 350 and have 50 mm face widths.

Gear	Number of Teeth
A	20
B	50
C	12
D	75



Find:

The normal load exerted by gear C on gear D and the speed of gear D and the safety factor for gear D based on bending and contact stresses.

F=4066 N; n_D=12.8 rpm; f_{sb}=2.76; f_{sc}=1