Mechanical Analysis and Design
ME 2104

Lecture 6

Modelling of a Tennis Ball Server

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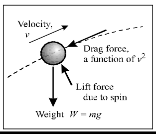
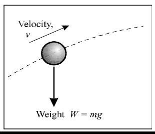
www.staff.city.ac.uk/~ra600/intro.htm

Modeling of a Tennis Ball Server

- Definition of a Projectile:
  - » A projectile is a fired, thrown, or otherwise projected object, such as a bullet, having no capacity for self propulsion
  - » In the absence of a propulsion force, the only force acting on an idealized projectile is gravity
  - » Additional forces – due to drag, lift and wind – will also be present and may or may not be significant





Modeling of a Tennis Ball Server

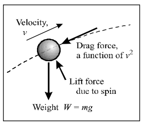
- Choice of Projectile:
  - » Neglecting drag, lift and wind should work well for typical projectile unless the problem involves a low mass or high cross-sectional area projectile
  - » Consider the drag force on a ping pong ball (immediately after launch; assuming a launch velocity of 9.5 m/s):

    $F_d = C_d \cdot \rho \cdot A \cdot v^2$

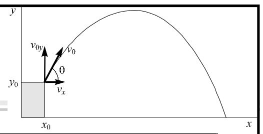
    F_d = (1/2)(1.21 kg/m3)[π(.038/2)^2](9.5 m/s)^2

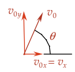
    = 0.062 kg·m/s^2 = 0.062 N
  - » Consider the force in light of the mass (0.0025 kg = 2.5 g) of a ping pong ball
  - » Using $F = ma$: 0.062 N = 0.0025·a

    a = 24.8 m/s^2 !! > 9.81 m/s^2



Basic Projectile Motion Equations



Commonly Used Projectile Equations (in *x-y* coordinate system)		
Coordinate Direction:	x	y
Acceleration:	$a_x = 0$	$a_y = -g$ where, g = 9.81 m/s^2
Velocity:	$v_x = v_{0x} = const$	(2) $v_y = v_{0y} - gt$
Position:	(1) $x = x_0 + v_x t$	(3) $y = y_0 + v_{0y}t - 1/2\, gt^2$
Additional equation:		(4) $v_y^2 = v_{0y}^2 - 2g\,(y - y_0)$
Calculating v_{0r} and v_{0s}, given v_0 and θ_0.		(5) $v_x = v_0 \cos\theta$ (6) $v_{0y} = v_0 \sin\theta$

Projectile Motion Calculation

Determining Values v_0 or θ needed to strike a target

Method A: Set your launcher at a fixed launch angle, θ. Calculate the launch velocity, v_0, needed to strike a target located at an arbitrary (x,y) position:

First write position equations:

(a) $x = x_0 + v_0(\cos\theta)t$ **(b)** $y = y_0 + v_0(\sin\theta)t - 1/2\, gt^2$

Let $L = (x - x_0)$ and $h = (y - y_0)$, solve (a) to get

(c) $v_0 t = \dfrac{L}{\cos\theta}$, sub this into **(b):** $h = \sin\theta\left(\dfrac{L}{\cos\theta}\right) - \dfrac{1}{2}gt^2$

Solve for t: (d) $\boxed{t = \sqrt{\dfrac{2(L\tan\theta - h)}{g}}}$

Substitute t into equation (c) and solve for v_0:

(e) $\boxed{v_0 = \dfrac{L}{\cos\theta}\sqrt{\dfrac{g}{2(L\tan\theta - h)}}}$

Projectile Motion Calculation

Determining Values of v_0 or θ Needed to Strike a Target

Method B: If your launcher can fire only at discrete values of v_0, select an appropriate v_0 and calculate the precise angle θ needed to strike a target located at (x,y): First write position equations:

(a) $x = x_0 + v_0(\cos\theta)t$ **(b)** $y = y_0 + v_0(\sin\theta)t - 1/2\, gt^2$

Let $L = (x - x_0)$ and $h = (y - y_0)$, solve (a) to get: **(c)** $t = \dfrac{L}{v_0\cos\theta}$

Substitute (c) into (b): $h = v_0\sin\theta\left(\dfrac{L}{v_0\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{L}{v_0\cos\theta}\right)^2$

Using $\sin\theta/\cos\theta = \tan\theta$ and the identity $(1/\cos\theta)^2 = \sec^2\theta = \tan^2\theta + 1$

We obtain **(d)** $\boxed{\tan^2\theta - \dfrac{2v_0^2}{gL}\tan\theta + \left[\dfrac{2v_0^2 h}{gL^2} + 1\right] = 0}$

This is a quadratic equation of the form $au^2 + bu + c$, where $u = \tan\theta$. The two roots correspond to the two launch angles ($\theta_1 = \tan^{-1}u_1$ and $\theta_2 = \tan^{-1}u_2$) which may be used to reach the same target.

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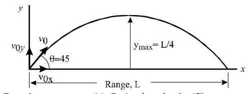
Projectile Example

Example 1: Projectile Heuristics

A "heuristic" is a "rule-of-thumb." The objective of this example is to use some simple numbers is to help the learner to more easily estimate approximately what speeds might be necessary for a launched projectile. Determine, for level ground, (1) The launch angle that gives the maximum distance; (2) The corresponding launch velocity; (3) The maximum height of the trajectory.



Equation summary: (a) Optimal angle: $\theta = 45^o$
(b) Minimum launch velocity to strike a target: $v_0 = \sqrt{gL}$
(c) Maximum height of the projectile: $y_{max} = L/4$

(1) To find the launch angle that gives the maximum distance (on level ground), first write position equations. (Let $L = (x - x_0)$ and $h = (y - y_0) = 0$):

(a) $L = v_0(\cos\theta)t$ (b) $h = 0 = v_0(\sin\theta)t - 1/2\,gt^2$

Divide (b) by t to get: (c) $0 = v_0(\sin\theta) - 1/2\,gt$

Rearrange (c) to get the landing time: $t_{land} = [(2v_0/g)\sin\theta]$

Substitute t_{land} into (a) to get $L = (2v_0^2/g)[\cos\theta \sin\theta]$

Since $\cos\theta\sin\theta = (\sin 2\theta)/2$, for level ground we can calculate the landing distance L from:

$L = (v_0^2/g)\sin 2\theta$

Since $\sin 2\theta$ is maximized at $2\theta = 90^o$, L is therefore maximum at $\theta = 45^o$. Specifically, $L = (v_0^2/g)$.

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Projectile Example

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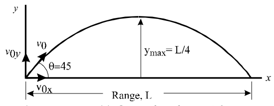

Equation summary: (a) Optimal angle: $\theta = 45^o$
(b) Minimum launch velocity to strike a target: $v_0 = \sqrt{gL}$
(c) Maximum height of the projectile: $y_{max} = L/4$

(2) The minimum launch velocity to reach a target (on level ground) at a distance L is:

$\boxed{v_0 = \sqrt{gL}}$. From $v_0 = \dfrac{L}{\cos\theta}\sqrt{\dfrac{g}{2(L\tan\theta - h)}}$.

(3) The maximum height, y_{max}, of the trajectory is found as follows: (Recall that at y_{max}, $v_y = 0$)

$v_y^2 = 0 = (v_0 \sin\theta)^2 - 2g(y_{max} - 0)$

Solve for y_{max}: $y_{max} = (v_0 \sin\theta)^2/2g$

If $\theta = 45^o$, $\sin 45 = \sqrt{2}/2$, and $(\sin 45)^2 = 1/2$. also note that $v_0^2 = gL$.

Finally, $\boxed{y_{max} = L/4}$

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Projectile Example

Example 1: Projectile Heuristics

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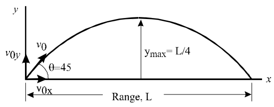

Equation summary: (a) Optimal angle: θ = 45°
(b) Minimum launch velocity to strike a target: $v_0 = \sqrt{gL}$
(c) Maximum height of the projectile: $y_{max} = L/4$

A simple numerical example: (These are "ballpark" numbers): A slow pitch softball pitcher wishes to hit home plate (approximately 15 m away) with a pitch with minimum effort. Assume no drag and that he releases the ball close to ground level (h = 0) very near the pitching rubber.

The optimal angle is 45°.

Minimum velocity $v_0 = \sqrt{gL} = \sqrt{9.81 \cdot 15} \approx \sqrt{144} = 12 m/s$.

The height of the trajectory would be $L/4 = 12/4 = 3$ m

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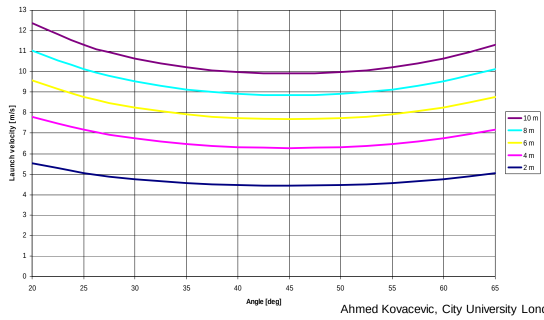
Projectile Example

Launch velocities vs. launch angles for selected target distances



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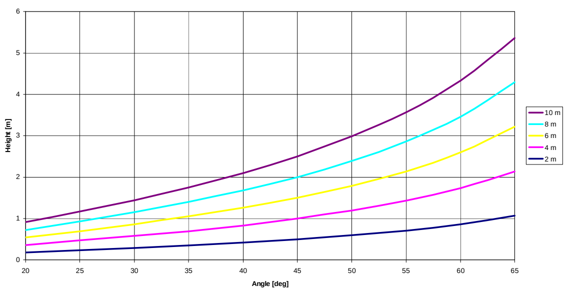
Projectile Example

Max height vs. launch angles for selected target distances



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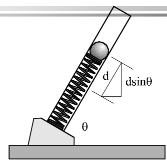
Work-Energy to Produce Desired V_0

The Work-Energy equation:

$\Sigma U_{1\text{-}2} = T_2 - T_1$

Where,

- T_1, T_2 – Initial and final Kinetic Energy of the projectile
- $T_1 = 0$
- $T_2 = mv^2/2$
- $\Sigma U_{1\text{-}2}$ - work done on the projectile while in the launcher.



Schematic of Launcher;
d = spring deflection
d sin θ = change in elevation of projectile

Recall: work = force x distance

Work done on the projectile is sum of :

a) Work of the spring: $U_s = +kd^2/2$
b) Gravity: $U_g = -mg\,(d \sin\theta)$
c) Friction – not estimated

$\Sigma U_{1\text{-}2} = T_2 - T_1$

$kd^2/2 - mg(d \sin\theta)$ – Friction = $1/2\, mv^2$ + KE (of spring, mechanism

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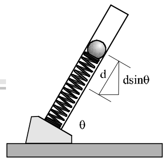
Work-Energy Example



Schematic of Launcher;
d = spring deflection
d sin θ = change in elevation of projectile

Determine spring stiffness, *k*

Determine deflection needed to strike an 8m target with a 55g projectile.

Neglect friction and kinetic energy imparted to mechanism.

(1) Spring Stiffness (Spring from a hardware store)
5kg load will compress the spring for 17.5 mm; Therefore, k_{sp} = 2.8 N/mm= 2.8 kN/m

(2) From previous chart: (v_0 = 9.2 m/s, θ = 56°) to strike 8 m target

(3) Given values:
(a) Mass = 55 g =0.055 kg
(b) Velocity = 9.2 m/s

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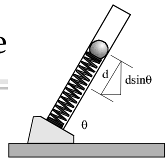
Work-Energy Example



Schematic of Launcher;
d = spring deflection
d sin θ = change in elevation of projectile

(4) Write work-energy equation:

$T_1+\Sigma U_{1\text{-}2} = T_2$

$kd^2/2 - mg\,(d \sin\theta) = mv^2/2$

$2.8\, d^2/2 - 0.055\ 9.81 \sin 56°\, d - 0.055\ 9.2^2/2 = 0$

This is a quadratic equation:
$1400d^2 - 0.447d - 2.327 = 0$, of the form $ad^2 + bd + c = 0$

Use the quadratic equation yields roots $d_{1,2}$ = 0.04 m

(5) Result:
The launcher spring needs to compress for 40 mm in order to propel a 55g projectile out of the launcher with a velocity of 9.2 m/s.
This is for a spring with k_{sp} = 2.8 N/mm.

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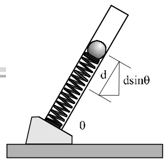
Work-Energy Example



(6) Caution
This is only approximate value since friction and kinetic energy of the launch mechanism are neglected. However, this gives an idea of how to select a spring or other mechanism for achieving launch of the ball.

(7) Velocity
To achieve accurate launch one needs to adjust velocity accurately.
If a launch to the distance of 8.5 m is required.
At the launch angle of 56° the required velocity is 9.48 m/s. The calculation yields a spring deflection of 41.5mm, only 1.5 mm more than needed for the first example.

Note: This is difficult to achieve!

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Summary

- For modeling purposes:
  - » Find range of velocities and angles necessary to get tennis ball to the targets
  - » Next, depending on your chosen design, determine how you will supply the necessary initial velocity
  - » Consider drag effects as well

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Plan for today

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