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Sometimes it does hurt to ask: The constructive role of articulating impressions

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ABSTRACT

Decisions can sometimes have a constructive role, so that the act of, for example, choosing one option over another creates a preference for that option (e.g., Ariely & Norton, 2008; Payne, Bettman, & Johnson, 1993; Sharot, Velasquez, & Dolan, 2010; Sherman, 1980). In this work we explore the constructive role of just articulating an impression, for a presented visual stimulus, as opposed to making a choice (specifically, the judgments we employ are affective evaluations). Using quantum probability theory, we outline a cognitive model formalizing such a constructive process. We predict a simple interaction, in relation to how a second image is evaluated, following the presentation of a first image, depending on whether there is a rating for the first image or not. The interaction predicted by the quantum model was confirmed across three experiments and a variety of control manipulations. The advantages of using quantum probability theory to model the present results, compared with existing models of sequence order effects in judgment (e.g., Hogarth & Einhorn, 1992) or other theories of constructive processes when a choice is made (e.g., Festinger, 1957; Sharot et al., 2010) are discussed.

1. Introduction

We can minimally define a choice as the process of selecting one alternative over another. A baseline intuition is that the values of these alternatives are subjectively represented prior to a choice, so that expressing a preference is the result of comparing these representations. However, it sometimes appears that the process of choosing one alternative over another alters their relative merits. For example, selecting a particular alternative appears to increase our preference for this option (e.g., Ariely & Norton, 2008; Kahneman & Snell, 1992; Payne, Bettman, & Johnson, 1993; Sharot, Velasquez, & Dolan, 2010; Sherman, 1980). Perhaps, this happens as a way to reduce uncertainty, second-thoughts or doubts in relation to the option which was not chosen (Festinger, 1957). In this work, we will propose and explore an innovative and surprising alternative approach to this issue, namely that there is a fundamental limitation in how uncertain information is represented (e.g., our preference for alternatives in relation to a particular choice). Then, a choice or judgment can be constructive, simply because of how potentialities regarding different options translate into a certainty for a particular option.

Clearly, the idea that judgments can be constructive is not novel. However, our examination of some well-known relevant findings, which are either presented as direct demonstrations of the constructive influence of decision making on preference or could perhaps be considered indicative of constructive influences, will show that our proposed experimental paradigm extends the circumstances under which

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it is thought that cognitive processes can be constructive. Specifically we explore whether just the process of articulating an impression for a stimulus (in the context of our paradigm, an affective evaluation), can have a constructive role, as evidenced by its impact on an evaluation of a subsequent stimulus.

The prediction that there might be constructive processes in the simple experimental paradigm we will shortly describe is motivated from recent work with quantum probability (QP) theory. QP theory is the theory for how to assign probabilities to events, from quantum mechanics, but without any of the physics (cf. Hughes, 1989; Isham, 1989). It is a formal theory of probability, just like classical probability (CP) theory. QP and CP theories are based on different axioms and so their predictions about probability assignment and inference can diverge. CP theory has provided an extremely influential framework for modeling cognitive processes, especially decision making (e.g., Griffiths, Chater, Kemp, Perfors, & Tenenbaum, 2010; Oaksford & Chater, 2007; Tenenbaum, Kemp, Griffiths, & Goodman, 2011). Of relevance presently is that QP theory, uniquely, embodies a formal component which can require a decision, judgment, or even expression of an impression to have a constructive role. On this basis, we develop a simple cognitive model, for the relevant empirical situation, based on QP principles and motivate a specific, a priori prediction. An empirical confirmation of this prediction will in turn support the QP principles at work, in relation to how a judgment (of any kind) can have a constructive role.

1.1. Going beyond relevant empirical evidence and the present paradigm

A cognitive process of decision making, judgment and so on can be said to be constructive, if the information on which the process operates is *altered*, as a result of the process. Order effects in tasks where discrete pieces of information are considered sequentially indicate constructive processes, but do not require them. For example, using a Gallup poll, Moore (2002) showed that American Vice President Gore would be rated less honest, if the previous question concerned the honesty of President Clinton and vice versa. Thus, the same judgment (is Gore honest) would be made differently, depending on the immediate context for the judgment (the previous question about Clinton). In fact, such question order effects are fairly common and are described in terms such as recency, primacy and contrast effects (Hogarth & Einhorn, 1992; Payne et al., 1993; Wang & Busemeyer, 2013). They arise both in terms of the relative order of answering questions (e.g., Moore, 2002) but also from the relative order of assessing evidence for a hypothesis, across a variety of domains, and indeed sometimes with participants expert in the relevant domain (e.g., Bergus, Chapman, Levy, Ely, & Oppliger, 1998; McKenzie, Lee, & Chen, 2002; for a review, see Hogarth & Einhorn, 1992).

Order effects in considering questions or the sequential assessment of evidence can be thought to reveal constructive processes, in the sense that, for example, the judgment about Clinton alters the information relevant to the judgment about Gore. The exact nature of constructive processes can be explained in various ways. For example, perhaps the first question activates thoughts, which subsequently affect consideration of the second question (Schwarz, 2007). However, such order effects would also be consistent with a process of weighting different pieces of information, depending on their order and other considerations (e.g., relative strength; Hogarth & Einhorn, 1992). Such weighting processes could operate on representations that are otherwise stable.

Generally, there has been controversy as to what exactly can be considered a constructive process (Brehm, 1956). A convincing demonstration that, at least in some cases, making a choice can be constructive, has been provided by Sharot et al. (2010). These investigators had participants select between two holiday destinations. After first rating how happy they would be at various destinations, participants then made a blind choice between destinations (they were told that the study concerned subliminal decision making). Subsequently they were informed which destination they had chosen, before participants again rated the destinations. The results showed a choice-induced change in preference and furthermore no such effect was observed when participants were given a choice from a computer.

Thus, a demonstration that choice is constructive would no longer be surprising. However, all existing empirical work on constructive processes and order effects concerns pieces of information or processes which are related to each other. For example, pieces of evidence for the same hypothesis (Hogarth & Einhorn, 1992) or a choice between alternatives that relate to the same objective (Sharot et al., 2010). Instead, in the present work, we focus on the extent to which just the process of expressing an impression for an image (providing a simple affective rating) can be constructive. Does the perception of a stimulus generate an impression, which can or cannot be read off, or does the process of expressing an impression have a constructive role, in relation to the underlying value? That such a process of articulating an impression can have a constructive role is novel and indeed seems more surprising, compared to putative constructive processes in choice.

Specifically, we propose to employ the following paradigm. We consider visual stimuli, which have a clear positive or negative affective content, and a task of providing affective ratings regarding this content. Methodologically, it is convenient to consider judgments relating to emotional content. We appear to have an ability to entertain positive and negative emotions concurrently (Brehm & Miron, 2006), e.g., in students' thoughts about graduation day or advertisements with mixed emotional appeals (Larsen, McGraw, & Cacioppo, 2001; Williams & Aaker, 2002). Especially for situations of emotional ambivalence, there is a reasonable intuition that a judgment can be constructive (clearly, it is not the case that a process of articulating an impression can be constructive in all cases: if you see a hammer and you state this, one hardly expects there to be a constructive process). These general ideas are the basis for the experiments.

In all three experiments, stimuli were hypothetical advertisements, which could have different affective content (positive vs. negative affective content). Consider presenting a positive advert, followed by a negative one. Previous studies suggest that the order of presentation can influence reactions (e.g., Ross & Simonson, 1991) and, indeed, such an effect of order can be easily anticipated within the broader context of the relevant literature (e.g., Hogarth & Einhorn, 1992; Moore, 2002). We can separate an order effect from an effect relating to the putative constructive role of judgments, with a simple design: we employ an identical order of presentation for two images and consider whether a rating or not for the first image can impact on the rating for the second image.

We employed a 2 × 2 within-subjects design with four conditions, involving the sequential presentation of positive and negative adverts. One factor was the order in which two (positive vs. negative) images were presented. In Experiment 1, in the positive–negative (PN) condition, an advert composed of a single positive image was presented, followed by a mixed advert, including the same positive image together with a negative image and vice versa for the negative–positive condition (NP). The second factor concerned the inclusion or exclusion of an intermediate judgment. The inclusion condition was matched in all respects with the exclusion condition, apart from the inclusion of an intermediate judgment for the first advert or not (Fig. 1). In the '*single rating*' condition, participants viewed the single image advert and then provided an affective evaluation for the mixed advert. In the '*double rating*' condition the same participants provided an intermediate rating to the single advert, before viewing and rating the mixed advert.

All the experiments match the structure of Experiment 1. With Experiment 1, we aim to establish the effect of interest: does the act of articulating an impression for the first image impact on the rating for the second image? Clearly, if there is an effect, it cannot be explained as an order effect. The motivation for Experiment 1 partly involved an assumption that emotional ambivalence is essential, before the measurement of an affective state can have a constructive role. In Experiment 2 we tested this assumption and found that measurements of affective states can be constructive, under a broader set of conditions. In Experiment 3 we replicated the main result of Experiments 1 and 2 with different materials and slightly different procedures. Experiment 3 also explored a possibility for the obtained results, based on anchoring. Overall, in all experiments we obtained the same main result, which shows that, when two stimuli are presented in identical orders, the presence of an intermediate affective judgment can impact on the last judgment. At the very least, this is a novel empirical demonstration, in relation to the kind of cognitive processes which can be constructive. Furthermore, we show how this result can be predicted fairly naturally, from basic QP theory principles, applied to a representation of the experimental task.

1.2. A QP theory model for constructive measurement

The simple empirical situation we consider can be modeled with QP principles, just on the basis of minimal assumptions about how the relevant information is represented and the impact of introducing the second advert. In general, cognitive QP representations are based on multidimensional vector spaces (called Hilbert spaces), in which the so-called state vector is assumed to correspond to the relevant cognitive state. In such spaces, different *subspaces* represent different possibilities for the cognitive state vector. For example, Fig. 2 shows a three dimensional overall space, such that the cognitive state vector represents the cognitive state of a hypothetical person, Sue. We consider two possibilities for Sue, whether she is happy (represented by a one-dimensional subspace; a ray) and whether she is unhappy (represented by a two-dimensional subspace, a plane; what determines the dimensionality of subspaces for different possibilities does not concern us presently). An important operation in QP theory is that of a projection: a projection takes a vector and 'lays it down' on a particular subspace (the projection is shown by the blue line in Fig. 2). One of the fundamental theorems of QP theory is that the

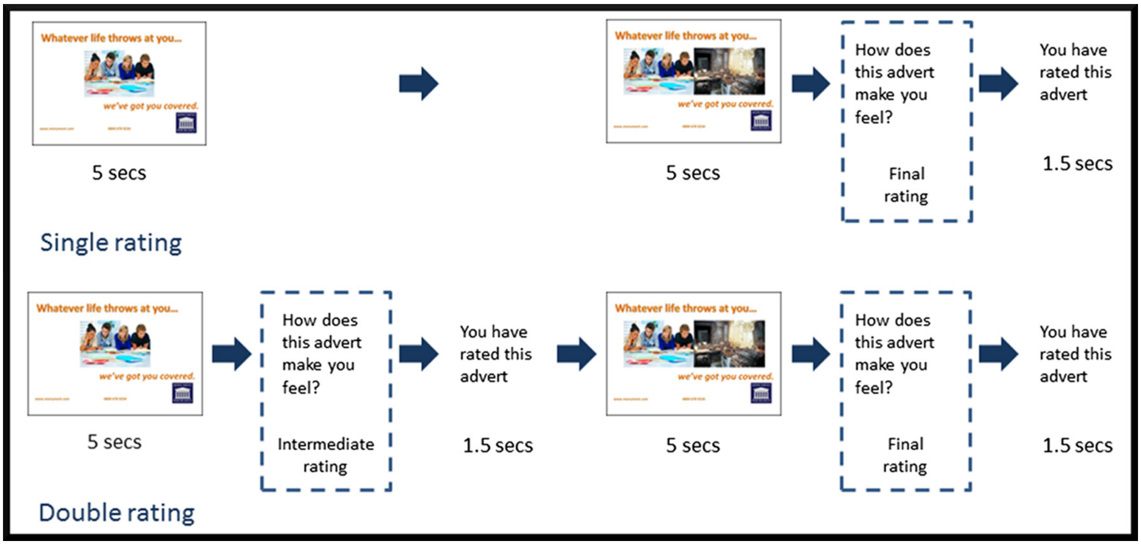

Fig. 1. Procedure for Experiment 1: sample advert used in PN condition and procedure for presentation of single and double rated adverts.

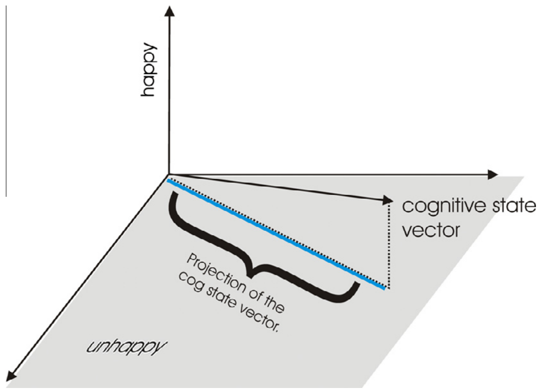

Fig. 2. An illustration of projection. The cognitive state vector is projected onto the two-dimensional plane (indicated by the shaded area), corresponding to the 'unhappy' possibility. The projection is denoted by the blue line and its length is the probability that the hypothetical person will decide she is unhappy. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

squared length of a projection, along a subspace, determines the probability that the corresponding possibility is true (of the system represented by the state vector). So, if we ask Sue whether she is unhappy, the squared length of the projection in the unhappy subspace is the probability that she will say yes. If she does say yes, then the cognitive state vector changes and is now a vector (of length one), along the projection (the blue line in Fig. 2).

QP cognitive representations can be understood mostly with the idea that greater projection (=overlap) between the state vector and a subspace implies higher probability. It is noteworthy that projection, as a modeling technique in cognition, has been discussed before outside QP theory (Sloman, 1993). A higher *consistency* between two possibilities implies a smaller angle between the corresponding subspaces, since, when the angle is small, it is easier to project from one subspace to the other. Likewise, mutually exclusive possibilities imply a 90° angle between the subspaces (as is the case for the happy, unhappy subspaces in Fig. 2). This is because when the state vector is along one subspace, e.g., the happy subspace, the projection to the unhappy subspace is zero (this is equivalent to saying that when $Prob(happy) = 1$ then $Prob(unhappy) = 0$).

Finally, in QP theory, in real spaces (i.e., in spaces of real numbers), dynamic processes can be modeled as *rotations* of the state vector (in general, such transformations are called unitary and they are the simplest kind of dynamical transformations employed in QP theory; cf. Asano, Ohya, Tanaka, Khrennikov, & Basieva, 2011). For example, suppose that Sue were to receive some information that would make her happy (e.g., her partner got her some nice flowers for her birthday). Then, the cognitive state vector would be rotated toward the happy ray, which would increase the projection on the happy ray, and so increase the probability that Sue will respond that she is happy, if she is asked.

Regarding the present paradigm, we develop a prediction from QP theory, in the simplest possible way (introductions to QP theory for cognitive psychologists can be found in Busemeyer & Bruza (2011), Pothos & Busemeyer (2013), Wang, Busemeyer, Atmanspacher, & Pothos (2013)). Thus, we restrict ourselves to an overall two-dimensional, real space, with different possibilities represented as rays.[1] An observer's cognitive state, in relation to the adverts in the task, is represented by a vector in this space, denoted as ψ. There is a set of rays for the possibility that the observer is in a purely positive/negative affective state (in relation to the adverts) and likewise there is a set of rays for the combined affective, perceptual impact of processing the positive or negative advert (Fig. 3A). Recall that ray proximity indicates consistency for the corresponding possibilities, so that, e.g., the ray for the Positive image is close to the one for Positive affect, because it is (trivially) assumed that seeing the positive advert is likely to generate an evaluation of positive affect (Fig. 3B; likewise for the Negative affect ray and the Negative image). Thus, the arrangement of rays relative to each other is automatic. Also, the two images were designed to be unrelated, so that the rays for each image are approximately orthogonal to each other, to mean that thinking of one image is unlikely to lead to thinking of the other.

In the PN condition, ψ starts in the positive image ray (corresponding to the positive advert; Fig. 3), since we assume the only influence on the cognitive state at that point is from processing the positive advert. In the PN single rating condition, the impact of introducing the composite positive–negative advert (Experiment 1) or just the negative advert (Experiments 2 and 3), is a fixed rotation toward the negative affect ray (Fig. 3C). This is an assumption that a stimulus does not produce an absolute affective reaction, but rather a fixed shift from the current state toward the rays for negative or positive affect. In other words, seeing a negative stimulus does not instantly put you in a negative affective state. Rather, it changes your affect in a *negative* direction, and your final affect will be a function both of the degree of change (represented by the rotation in the quantum model) and your initial affective state. Such an assumption is intuitive, but also appears in various forms across a range of psychological theories (e.g., Hogarth & Einhorn, 1992; see also relative judgment models of perceptual differences, Laming, 1984; Stewart, Brown, & Chater, 2005).

After the second advert is presented, the observer is asked to rate it. The projection onto, e.g., the negative affect ray will be greater, if the angle between ψ and the negative affect ray is smaller. Greater projection onto the negative affect ray indicates greater probability for a negative rating, which we assume translates to a more negative rating, on average, across participants (Fig. 3D).

This is the key aspect of QP theory needed for the present model: unless the cognitive state vector is wholly within the positive or negative affect ray, then the observer is in a *superposition* regarding his/her affective evaluation of the adverts. Superposition is a technical term in QP theory. A superposition state between positive and negative affect

[1] The judgements made in the present experiments are ratings on a nine-point scale and, strictly speaking, should be represented by (at least) a nine-dimensional vector space. However, for simplicity we have used a two-dimensional vector space, so that judgments can be considered as either of positive or negative affect. We believe that this approach does not reduce the generality of our ideas and it does greatly simplify their exposition.

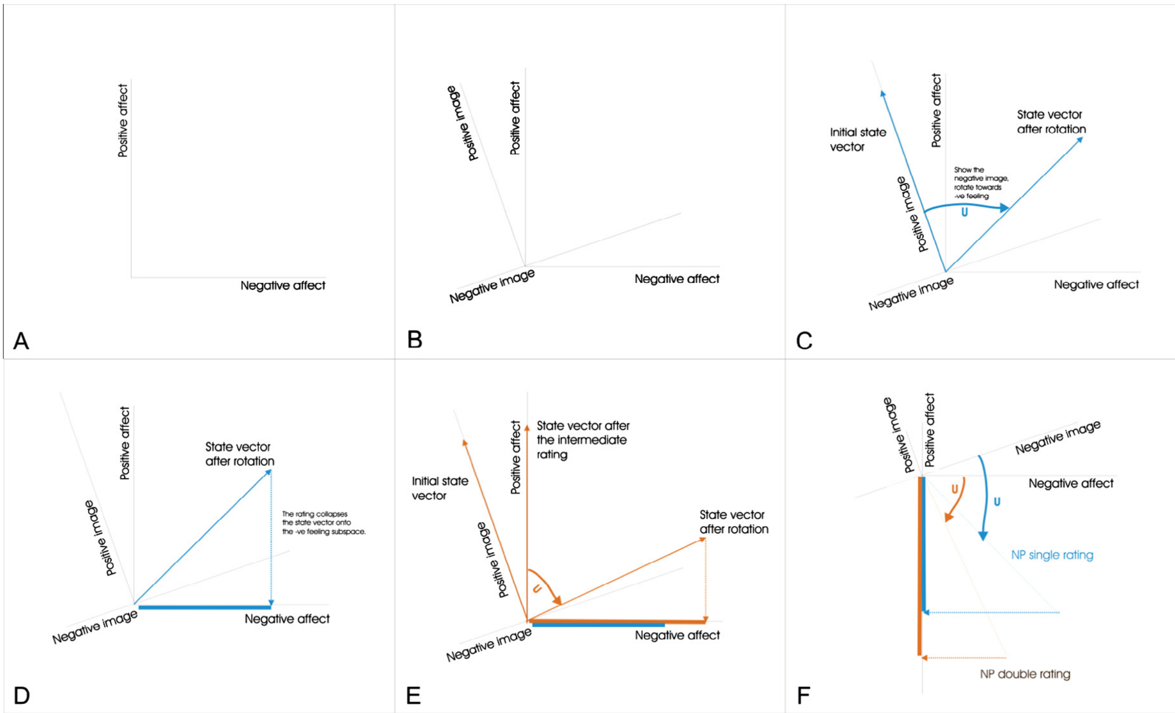

Fig. 3. Quantum Probability Model: a QP model for the constructive role of measurement in the present experiments, in the PN condition (A–E) and NP condition (F).

means that there is a potentiality for both evaluations (i.e., a non-zero probability of making either evaluation). But, it also means that the state is not consistent with either evaluation, prior to making a judgment (superposition states sharply contrast with classical linear mixtures, which encode uncertainty in the existing value of the state). Once a judgment is made, the cognitive state vector transitions to the ray consistent with the result of the judgment – this is the aspect of QP theory that is constructive. Because the negative adverts were designed to elicit, fairly unequivocally, negative feelings, and likewise for the positive adverts, we assume that judgments consistently lead to the expected transitions. (In practice, in Experiment 1 four participants, out of 54, were excluded from the results because their responses were not consistent with expectation. For experiments 2 and 3 all responses led to the expected judgment.)

So, in the double rating condition, with the intermediate rating, for the first advert, the observer forces a transition of the cognitive state vector to the positive affect or the negative affect ray, depending on the advert. This is an extra change to the state vector, which compounds with the change from introducing the second advert (Fig. 3C vs. Fig. 3E).[2] For example, in the PN condition, having the intermediate rating moves the cognitive state vector closer to the negative affect ray, making a negative rating more likely, compared to not having it. Overall, in the PN condition, the intermediate judgment makes the second advert appear more negative, and exactly vice versa for the NP condition (Fig. 3F). Psychologically, the QP model prediction can be interpreted in the following way. In the PN order, for example, how negatively the second advert is perceived will depend on the contrast between the second advert and the cognitive state prior to its introduction. Without the intermediate rating, this cognitive state will correspond to the P advert, so the perception of the N advert will depend on the contrast between the two. With the intermediate rating, we assume that the P advert is rated positively, so the cognitive state changes to correspond to positive affect (this is the constructive step in the QP model). Then, the perception of the subsequent N advert will depend on the contrast between positive affect and the N advert. This would make more obvious the fact that the second advert is negative, hence leading to a more negative rating.

In sum, the QP theory prediction arises fairly naturally, from straightforward assumptions about the representation of the various components in the task (that is, the stimuli and the corresponding affective evaluations). One psychological assumption concerns the impact of introducing the second stimulus (i.e., a fixed rotation, relative to the initial state; Hogarth & Einhorn, 1992; Laming, 1984; Stewart et al., 2005).

Other QP approaches to decision making are based on the same core principles of QP theory and, specifically,

[2] A consistency consideration determines the direction of rotation. In e.g., the PN case, we want the impact of introducing the negative image to be always (at least, with respect to plausible conditions in the experiment) a rotation *away* from the positive image ray. So, in the PN double rating condition, a rotation away from the positive image ray and toward the negative affect ray has to be clockwise and, by consistency, we assume the same rotation direction in the single rating case. Clearly, a more general experimental paradigm would require more general modeling assumption, but the present approach suffices for the simple experiment we ran.

share the key assumption employed presently, that a decision must involve a corresponding projection of the state vector (e.g., Aerts, 2009; Asano, Basieva, Khrennikov, Ohya, & Tanaka, 2012; Asano, Ohya, Tanaka, Basieva, & Khrennikov, 2011; Busemeyer, Pothos, Franco, & Trueblood, 2011; Pothos & Busemeyer, 2009; Yukalov & Sornette, 2010).[3] Note that different QP models do somewhat differ in the psychological assumptions which they embody, over and above core quantum principles. For example, Busemeyer et al.'s (2011) model for the conjunction fallacy (Tversky & Kahneman, 1983) assumes that, when evaluating conjunctions of incompatible possibilities, the more likely one is evaluated first. In the present model, we had to introduce some assumptions regarding dynamics. In Pothos and Busemeyer's (2009) prisoner's dilemma (Shafir & Tversky, 1992) model, decision makers are assumed to be driven by a prerogative to maximize gain and by a cognitive dissonance bias. In order to create cognitive theory, we think it is inevitable that the principles of any mathematical formalism, such as QP theory, would need to be augmented by additional assumptions, regarding the underlying psychology. Of course, there is a need for consistency between different approaches in terms of mathematical operations (as there is for QP models; cf. Jones & Love, 2011).

2. Experiment 1: the influence of an intermediate evaluation on mixed adverts

We chose the first stimulus as a single image advert and the second as that image augmented with another image of opposite affect to create a mixed advert. Mixed adverts were employed, as we thought some ambiguity regarding the affective evaluation of the last advert may amplify the perhaps small interaction we were looking for. As noted, previous work suggests there should be a recency effect (Trueblood & Busemeyer, 2011), but this is not the empirical result of interest. Rather, we aim to examine the impact of an intermediate measurement, in identical stimulus presentation orders.

2.1. Method

2.1.1. Participants and design

Fifty-four Swansea University students participated for course credit (45 women, average age 21.74 years). We employed a within-subjects design with two independent variables: advert order (PN, NP) and rating (single, double).

2.1.2. Stimuli

Adverts were designed, so that having a positive and negative image together in the same advert would make sense. Different products were used for the PN condition (insurance; Fig. 1) and the NP condition (smartphone; Fig. 4), so as to avoid interference between conditions. For the PN set three positive adverts were individually presented, and three mixed adverts, with each of the positive images joined with a negative one, and analogously for the NP set. Intended affective response and unrelatedness were confirmed in a pilot (see Appendix A). The adverts were randomly presented with 24 filler adverts for a camera.

2.1.3. Procedure

Participants first completed a six-item current mood questionnaire. They were then told that they would see several adverts and that for each advert, when asked, they should answer the question 'how does this advert make you feel?', responding on a nine-point scale, with anchors "1: very unhappy to 9: very happy". Each trial involved the presentation of a single image advert, followed by a request for rating (double rating condition) or not (single rating condition), followed by the mixed advert and a final request for rating (Figs. 1 and 4). Trials were organized into two blocks. One block contained the six single rating PN adverts and six double rating NP ones, together with 12 filler adverts (also rated). The other block contained the same adverts, but switching the requirement for single vs. double rating (i.e., participants rated adverts twice, once in the single rating condition, once in the double rating one). In all experiments, block order was counterbalanced and trial order within blocks randomized. That is, approximately half the participants first went through the block with single rating adverts only and then the block with double rating adverts, and vice versa for the remaining participants. The advantage of this design is that we can assess the putative impact of the intermediate trial, in double rating adverts, relative to matched single rating adverts, using a within participants design.

2.2. Results

As the valence of the images had been established in the pilot study, we excluded four participants whose ratings for the single image adverts were over one standard deviation below (positive adverts) or above (negative adverts) the mean.

We conducted a two (advert order: PN, NP) × two (rating: single, double) repeated measures ANOVA on participant ratings for the second, mixed adverts (Fig. 5). There was a main effect of advert order ($F(1,49) = 7.98$, $p = .007$), but not of rating ($F(1,49) = 0.04$, n.s.). Importantly, the advert order × rating interaction was significant ($F(1,49) = 10.96$, $p = .002$). Paired samples t-tests showed that, in the PN condition, with an intermediate rating, ratings for the second adverts ($M = 4.04$, $SD = 1.17$) were significantly lower (i.e., the ratings were more negative), than those without the intermediate rating ($M = 4.34$, $SD = 1.43$; $t(49) = 2.18$, $p = .02$, (all t-tests are two tailed); $d = .31$). In the NP condition, with an intermediate rating, ratings for the second adverts were significantly higher (i.e., the ratings were more positive; $M = 4.94$, $SD = 1.21$), than without the intermediate rating ($M = 4.60$, $SD = 1.22$; $t(49) = -2.39$, $p = .01$; $d = .34$).

[3] The dynamics in most QP decision making models, including the present one, are the so-called unitary dynamics, which are applicable when one can assume limited or no interactions of the system of interest with the environment. Open system dynamics relax this assumption (e.g., Asano, Ohya, Tanaka, Basieva et al., 2011; Asano, Ohya, Tanaka, Khrennikov et al., 2011). In some cases of QP cognitive models, unitary dynamics can be thought of as an approximation to the more realistic (but also technically more complex) open system dynamics; our use of unitary dynamics presently should be approached in this way.

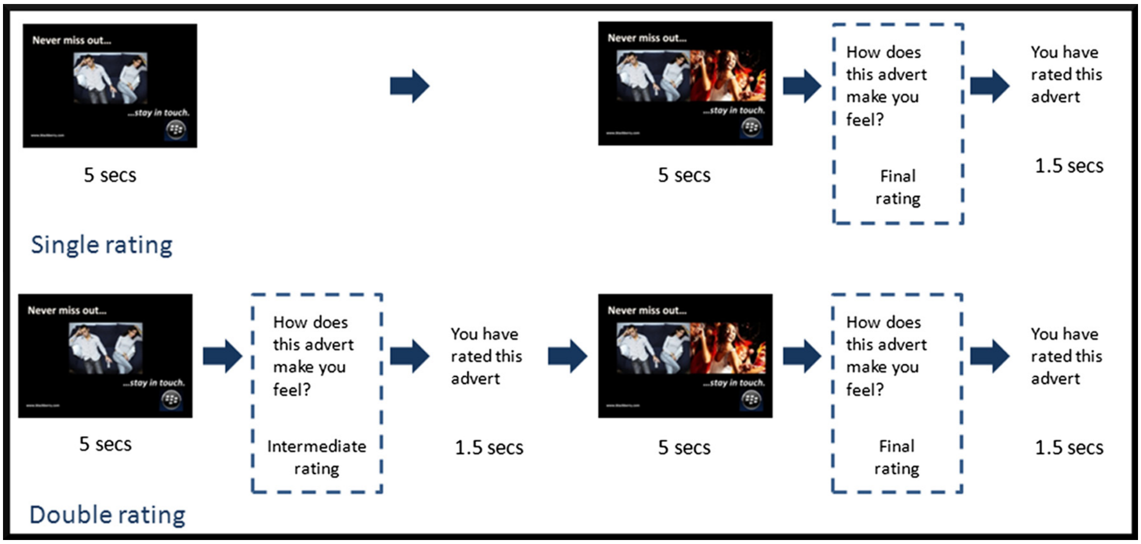

Fig. 4. Procedure for Experiment 1: a sample advert used in the NP condition and the procedure for presentation of single and double rated adverts.

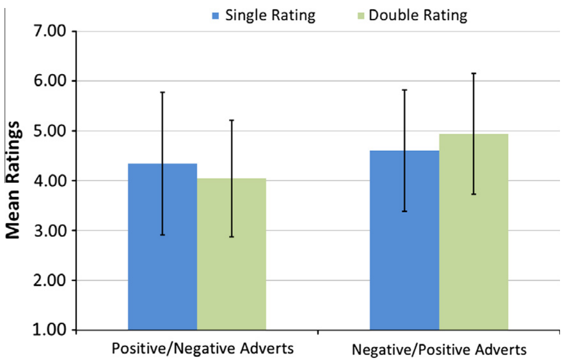

Fig. 5. Experiment 1 Results: mean participant ratings of single and double rated PN and NP adverts (error bars represent standard deviations). Note that, here and elsewhere, the max possible value for the mean ratings variables is 9, but this value was never observed.

2.3. Discussion

In otherwise matched conditions, with an intermediate rating, the rating of the second advert was more negative in the PN condition and more positive in the NP condition. We outlined a model for the impact of the intermediate rating, based on QP theory. The advantage of the model is that it can provide a specific prediction regarding the direction of the interaction, with fairly minimal assumptions, primarily about the relative positioning of the components in the experimental task (the rays corresponding to affect and the images) and the impact of introducing a second stimulus on the cognitive state. The key theoretical characteristic of the QP model is that measurement is required to be constructive (unless the cognitive state is already fully consistent with one of the possible outcomes of the measurement). Thus, the consistency between the QP model prediction and the empirical result can be taken as an indication that the intermediate rating has a constructive influence on the cognitive state, which is evident in the rating for the second advert.

The design of the first experiment was partly based on the assumption that a putative constructive role for measurement is more likely to be observed in situations of emotional ambivalence, that is, situations (stimuli) which lead to both positive and negative affective reactions (as we expected to be the case for the second advert). This determined our decision to augment the first image with another one of opposite affect (whether the first or the second advert impacts the most on the final rating is irrelevant for our purposes; note, there was some evidence for a recency effect; e.g., Trueblood & Busemeyer, 2011). In Experiment 2, we challenge this assumption and ask whether the same interaction, based on the presence of the intermediate judgment or not, can be observed, when a positive advert is followed by (just) a single (as opposed to mixed) negative advert, or vice versa. The advantage of such a design is that it can help eliminate an alternative explanation for the results of Experiment 1, which does not require a constructive role for measurement. In Experiment 1, perhaps the inclusion of the first image in the later mixed advert served as a reminder of whether a rating for the first advert had been provided or not, so influencing the rating of the mixed advert. If in Experiment 2, when a positive (negative) advert follows a completely new negative (positive) one, we still observe the same interaction as in Experiment 1, this alternative explanation cannot apply.

Note, finally, that the prediction from the QP model is effectively unchanged. The model we outlined needs to be modified in only one way, namely that the rotation as a result of introducing the second advert is greater, since the second advert would more clearly lead to negative or positive affect. The prediction for the interaction remains unchanged.

3. Experiment 2: the influence of an intermediate evaluation on single image adverts

3.1. Method

3.1.1. Participants and design

Twenty Swansea University students participated in the experiment for course credit (15 women, average age 20.1 years). The design was as for Experiment 1.

3.1.2. Stimuli

Positive and negative images from Experiment 1 were used together with new images, piloted and selected as before. Realistic-looking adverts were created, which always included a single positive or negative image (see Fig. 6). Unlike Experiment 1, both products were used in both conditions: 12 adverts in the PN condition included three positive insurance, three negative insurance, three positive smartphone and three negative smartphone, and likewise for the NP condition. The images were randomly presented with 24 filler adverts.

3.1.3. Procedure

The procedure was as in Experiment 1, except that all adverts included single images. Trials were organized into two blocks. One block contained the six single rating PN smartphone adverts, six double rating PN insurance adverts, six single rating NP insurance adverts and six double rating NP smartphone adverts. The other block contained the same adverts, but switching the requirement for single vs. double rating. Block order was counterbalanced and trial order within blocks was randomized.

3.2. Results

We conducted a two (advert order: PN, NP) × two (rating: single, double) repeated measures ANOVA on the ratings for the second adverts (Fig. 7). There was a main effect of advert order ($F(1,19) = 117.04$, $p < .001$), but not of rating ($F(1,19) = 2.88$, n.s.). As before, the advert order × rating interaction was significant ($F(1,19) = 63.12$, $p < .001$). Paired samples t-tests showed that, with the intermediate rating, the second advert was rated more negatively in the PN condition, compared to without the intermediate rating ($M = 3.81$, $SD = 0.71$ vs. $M = 4.36$, $SD = 0.69$; $t(19) = -4.58$, $p < .001$; $d = 1.03$) and the positive advert was rated more positively in the NP condition ($M = 6.63$, $SD = 0.73$ vs. $M = 5.60$, $SD = 0.75$; $t(19) = 4.78$, $p < .001$; $d = 1.06$).

3.3. Discussion

Experiment 2 produced the interaction predicted by the quantum model and observed in Experiment 1. Moreover, this replication provides a stronger test, as both adverts were now associated with fairly clear positive or negative affect.

Experiment 3 was designed with two purposes in mind. First, we sought to replicate the previous results, with different, better controlled stimuli. Specifically, if a pair of positive–negative images share some elements, the affective contrast may be more pronounced. So, as a methods manipulation, we created positive–negative image pairs to either share or not share a main element, to examine whether this is indeed a factor which possibly moderates the effect of interest, or not. We also designed adverts using a database which contained images whose valence had been externally validated (Geneva Affective Picture Database (GAPED): Dan-Glauser & Scherer, 2011).

Second, we examined yet another possibility for the source of the interaction observed in Experiments 1 and 2, alternative to the one implicated in the QP model (namely, that judgment is constructive). Specifically, what drives the observed result is possibly the availability of a rating (whatever the source) after the first advert, rather than the act of measurement by the judge about his or her own feelings. The effect of a rating from an independent

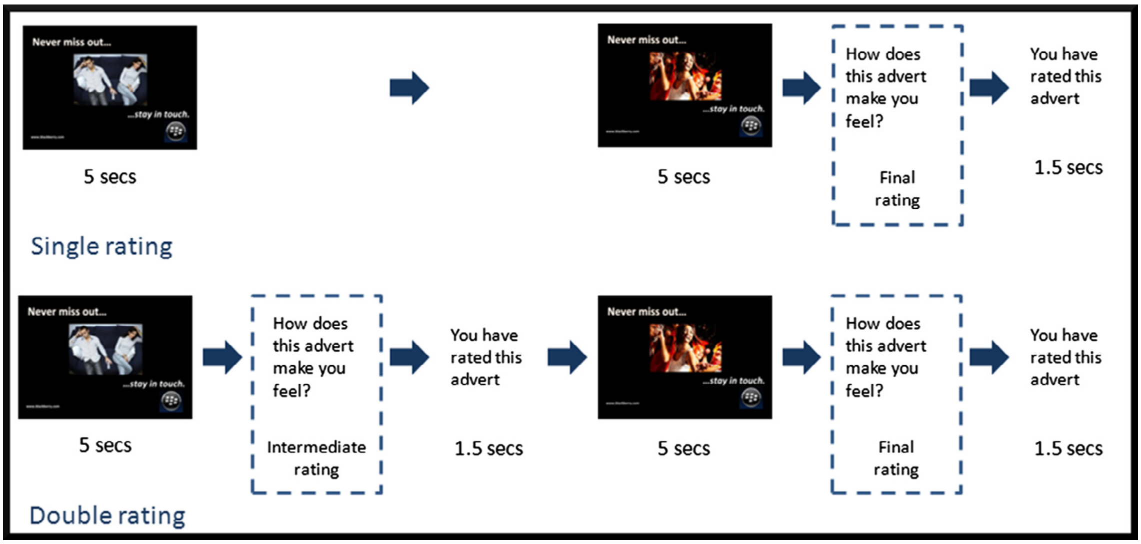

Fig. 6. Procedure for Experiment 2: a sample advert used in the NP condition and the procedure for presentation of single and double rated adverts.

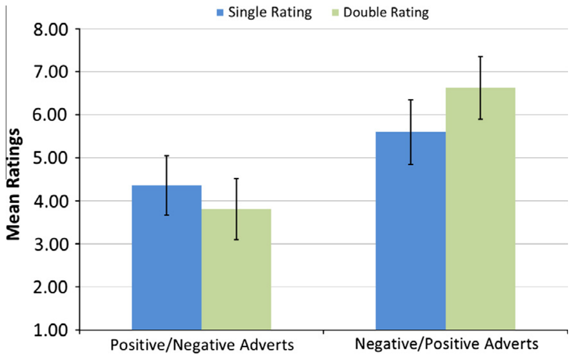

Fig. 7. Experiment 2 Results: mean participant ratings of single and double rated PN and NP adverts (error bars represent standard deviations).

source is a possibility consistent with an idea from the anchoring and adjustment model, whereby people anchor onto an initial value, which is then adjusted to produce a more exact estimate (Hogarth & Einhorn, 1992; Tversky & Kahneman, 1974). Also, in Tversky & Kahneman's (1974) research, and other analogous studies, it has been shown that when people are first asked to make a comparative judgment (e.g., is the percentage of African countries in the UN higher or lower than 25?), followed by an absolute judgment (e.g., what is the exact percentage of African countries in the UN?), the latter judgment is biased toward the comparison value provided in the first judgment, even if that initial value is randomly generated. Although in our experiments ratings for both stimuli are (more) precise, it seems plausible to argue that a more readily accessible rating for the first advert is perhaps an anchor, which helps generate a contrasting rating for the second advert. In Experiment 3 we explored this idea by showing participants a randomly generated rating for the first stimulus, before they rated the second stimulus, and also provided some additional controls.

4. Experiment 3

4.1. Method

4.1.1. Participants and design

Forty-one Swansea University students participated for course credit (37 women, average age 19.72 years). There were three within-subjects independent variables: advert order (PN, NP), rating (single, double, control) and advert content (shared element, not-shared). We intended that in some of the trials, a rating would be provided to participants and a between-subjects manipulation explored two possibilities for doing so (control type: random participant rating vs. computer rating; Fig. 8).

4.1.2. Stimuli

An equal number of adverts for a camera and a geographical magazine were created, each one including a single positive or negative image. Images were drawn from the GAPED (Dan-Glauser & Scherer, 2011) and selected for their valence and relevance to advert themes. There were 16 positive and 16 negative adverts. Half the positive/negative adverts included a shared main element, in that the same subject was shown in different circumstances (e.g., a stag on a mountain and a stag that had been shot; see Fig. 8) and the other half had different main elements (e.g., sleeping gerbils and caged geese; see Fig. 8).

4.1.3. Procedure

The procedure was as previously for single and double rated adverts. Regarding the between-subjects control manipulation, after the presentation of the first advert, 21 participants were shown a rating of an allegedly random participant and asked to confirm the rating by pressing the appropriate key. The provided ratings were randomly generated and ranged from two to eight, to avoid unrealistic extreme values. The act of telling participants that an intermediate rating had been provided may somehow influence the evaluation of the second advert. Thus, the remaining 20 participants were simply told that the computer had rated the advert but were not told the rating. So, all participants were tested with one of the control manipulations (random participant rating or computer rating), as well as the main experimental manipulations.

Trials were organized into four blocks. Each block contained four single rated trials, four double rated trials and four control condition trials, using either eight PN and four NP adverts or four PN and eight NP adverts. Block order was counterbalanced and trial order within blocks was randomized.

4.2. Results

We conducted a two (advert order: PN, NP) × three (rating: single, double, control) × two (advert content: shared element, not-shared) × two (control type: random participant rating, computer rating) mixed ANOVA on participants' ratings for the second adverts. We first examined the advert content manipulation. There was a significant main effect of advert content on ratings ($F(1,39) = 10.75$, $p = .002$); regardless of whether an advert included a positive or negative image, adverts designed so that, pairwise, they shared a main element were overall rated lower, than ones not sharing a main element ($M = 4.11$, $SD = 2.09$ vs. $M = 4.39$, $SD = 2.08$). Importantly, there were no interactions between advert content and any of the other factors, so that, in what follows, the advert content factor was ignored.

We next examined a possible anchoring effect. Is it the case that just the availability of a rating for the first advert influences the second advert rating? We first considered participants' own ratings for the first and second stimulus in the double rating condition. There was no evidence for an anchoring effect, as there were low, non-significant correlations between participant ratings for the first and second advert in the PN ($r = -.36$, n.s.) and NP ($r = .26$, n.s) conditions. Note, the same result is also replicated in Experiment 1 (PN: $r = .26$, n.s.; NP: $r = .18$, n.s.) and Experiment 2 (PN: $r = .40$, n.s.; NP: $r = .02$, n.s.). Next, for the control type factor (comparing participants who, for the first image, saw either a random participant rating or were told that there was a computer rating), there was no significant main effect

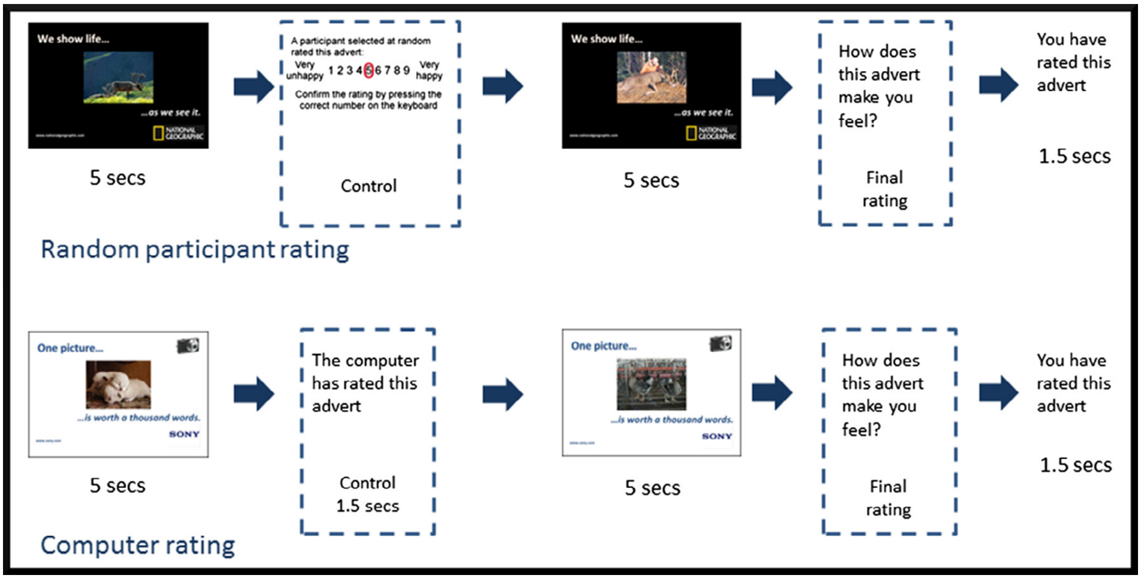

Fig. 8. Control manipulations in Experiment 3: a sample advert used in the PN condition and the procedure for presenting the control adverts.

on the rating for the second image ($F(1,40) = 0.45$, n.s.). Finally, with the random participant rating, we also found low, non-significant correlations between the random rating and the participant rating for the second advert in the PN ($r = .06$, n.s.) and NP ($r = -.01$, n.s) conditions. So, the control factor was ignored in all further analyses. We next explored whether the effect of interest replicates with the new stimuli in Experiment 3 (Fig. 9).

Mauchly's test indicated a violation of sphericity for the main effect of rating, $\chi^2(2) = 13.54$, $p < .05$ and the advert order × rating interaction, $\chi^2(2) = 25.03$, $p < .001$. Therefore, degrees of freedom were appropriately adjusted. There was a main effect of advert order ($F(1,39) = 468.08$, $p < .001$). PN ordered adverts ($M = 2.46$, $SD = 0.72$) were rated lower (more negatively) than NP adverts ($M = 6.04$, $SD = 1.16$). There was a significant main effect of rating ($F(1.54,60.01) = 12.54$, $p < .001$). Importantly, the advert order × rating interaction was significant ($F(1.35,52.61) = 18.86$, $p < .001$). Paired samples *t*-tests confirmed that in the PN condition, with the intermediate rating, the second advert was rated more negatively than without ($M = 2.60$, $SD = 0.81$ vs. $M = 2.43$, $SD = 0.63$; $t(40) = -1.92$, $p < .05$, $d = .37$). In the NP condition, the intermediate rating resulted in the second advert being rated more positively ($M = 5.52$, $SD = 1.22$ vs. $M = 6.29$, $SD = 1.12$; $t(40) = 4.51$, $p < .001$; $d = .67$).

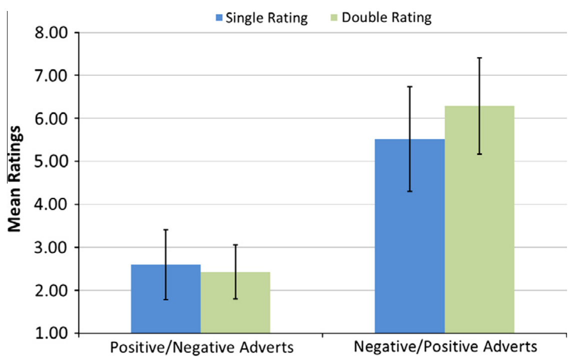

Fig. 9. Experiment 3 Results: mean participant ratings of single and double rated PN and NP adverts (error bars represent standard deviations).

4.3. Discussion

Does the intermediate rating affect the rating for the second image because it generates an anchor on which to base the second rating (Tversky & Kahneman, 1974)? We found no evidence for this possibility, either on the basis of correlating participants' ratings for the first and second image in the double rating trials or using the control condition, whereby participants were shown the rating from a hypothetical participant.

The other objective of Experiment 3 was to replicate the key interaction observed in Experiments 1 and 2, with novel stimuli, which was accomplished. The stimuli of Experiment 3 were also designed to incorporate a manipulation regarding thematic content. It is possible that the affective contrast between two sequentially presented stimuli would be greater, if they share the same theme. If that were to be the case, then this would warrant a closer examination of the results across all experiments, but this thematic content factor was not significant.

5. Alternative theory

5.1. Can the anchoring and adjustment model and other relevant theory account for the results generated by the proposed paradigm?

We have argued that a QP approach provides a fairly natural perspective on how to model putative constructive processes in cognitive processing. Nevertheless, the consideration of constructive processes in psychology (theoretically and empirically) has a long history. In this section we consider whether there is alternative theory, which could potentially account for the empirical results we reported.

There are two classes of relevant theory, (a) theories which incorporate assumptions about constructive processes directly and (b) theories which can predict order effects, without necessarily assuming constructive processes. In the first category, an influential example is Fuzzy Trace Theory (FTT; Reyna, 2008; Reyna & Brainerd, 1995), according to which memory representations have two components, verbatim and gist. The verbatim component corresponds to a veridical record of the encoded information and the gist one is a corresponding summary of the main semantic characteristics. The gist component can depend on contextual influences, the observer's perspective, and interpretative biases. Therefore, generating a gist representation of some information is a constructive process (Brainerd, Reyna, & Ceci, 2008).

Relatedly, in decision making, Query Theory (Johnson, Haubl, & Keinan, 2007) is based on an assumption that value is constructed, depending on the task at hand, for example, whether a good is being offered or chosen (as relevant in the endowment effect; Kahneman, Knetsch, & Thaler, 1991). Query Theory assumes that value is generated from a series of internal questions regarding characteristics of the relevant good, so that, depending on query order, the output from different queries may interfere.

Order and context effects are postulated in many kinds of theories in decision making, but without an assumption of constructive processes. For example, in Birnbaum's (2008) configural weight models, the evaluation of gambles will depend on various contextual influences, including rank order of a gamble against compared alternatives, and the perspective of the decision maker (e.g., whether something is sold or bought). Also, Anderson's (1971) Integration Theory is a family of models, which aim to explain the process of integrating information from various sources, to form an overall judgment. Each piece of information has a weight associated with it (which can depend on e.g., order or context), which can alter the impact that the information has on the overall judgment.

The implications from such work (see also Festinger, 1957; Schwarz, 2007) for the proposed paradigm are limited. Part of the issue is that the task employed in the current experiments was very simple: there was no complex information to be recalled or complex decisions to be reached; participants provided a simple affective evaluation. Also, the putative constructive impact on the final rating from the intermediate one cannot be explained as an order effect (the order was the same across the critical conditions; we compared the rating for the second image, depending on whether the first rating was made or not).

A framework which can provide quantitative predictions for the present results is Hogarth and Einhorn's (1992) anchoring and adjustment model, which is a model for how evidence is integrated to form an opinion about a hypothesis. The appeal of the model is that it does not just take into account order effects, but also the potential impact of intermediate evaluations. Note that the empirical situation we considered was about the impact of an earlier judgment on a later, similar judgment, but toward a different stimulus (in Experiments 2 and 3; in Experiment 1 the second stimulus was related to the first). Thus, the different stimuli do not correspond to different pieces of evidence, for the same hypothesis and, so, potentially Hogarth and Einhorn's (1992) theory is not applicable. Nevertheless, the computations in Hogarth and Einhorn's model can apply to the present empirical situation, so it becomes interesting to explore the predictions.

Recall, the main finding is that, with the intermediate rating, the negative advert was judged more negatively in the PN condition and the positive advert was judged more positively in the NP condition. In other words, the intermediate rating appears to increase *recency*, in relation to the condition without the intermediate rating (note, in decision making, recency is typically defined as an increased impact of the last item on a final evaluation, in relation to the studied and converse order, which is a little bit different to what we have here). But, Hogarth and Einhorn (1992) note (p.6) that there is "primacy in 19 of 27 EoS studies and recency in 16 of 16 SbS studies." In their terminology, EoS means an End of Sequence mode of assessing evidence, whereby evidence is assessed altogether, after all pieces of evidence have been presented. SbS means Step by Step, so that the impact of each piece of evidence on a hypothesis is assessed as soon as it is presented. An SbS mode of evaluation would imply, in our terminology, intermediate judgments. Thus, according to Hogarth and Einhorn's (1992) fairly comprehensive (at the time) review, intermediate judgments appear to lead to recency, which means that the last item should have a greater impact on the final weight. To reiterate, our key result is that with intermediate judgments there was also a (sort of) recency effect (e.g., if the last item is negative, the intermediate judgment makes it look more negative).

However, this impression of consistency is misleading. The bulk of results Hogarth and Einhorn (1992) consider concern assessing the order of several pieces of evidence. Indeed, Hogarth and Einhorn (1992) defined 'short' series of evidence to correspond to 2–12 items (p.6) and long to correspond to 17 items or more. In their own experiments, in the conditions in which the lowest number of items was employed, Hogarth and Einhorn (1992) used an initial description (which can generate an initial belief, regarding the hypothesis of interest), with at least two additional pieces of evidence. It therefore becomes pertinent to explore in more detail the implications of Hogarth and Einhorn's (1992) framework for the present experimental paradigm. It is worth noting that Hogarth and Einhorn (1992) themselves acknowledge that their framework is descriptive (p. 2). So, even if the predictions from Hogarth and Einhorn's approach were to be consistent with our empirical results, it would be worth seeing whether an explanation based on formal principles (from QP theory) is possible (similar ideas are often expressed, e.g., Jern & Kemp, 2013). But this is not the case. Appendix B provides a thorough analysis of Hogarth and Einhorn's (1992) model, which shows that their model cannot predict an influence from the intermediate rating. Briefly, the problem is that the empirical paradigm we consider is so simple, that many of the interesting effects otherwise predicted within Hogarth and Einhorn's (1992) framework disappear.

5.2. Constructive processes in CP theory

CP theory is probably the most dominant computational/mathematical research tradition currently used in cognitive psychology (Griffiths et al., 2010; Tenenbaum et al., 2011). For example, since Wason's (1960) results showing that naïve observers do not appear to reason in a way consistent with classical logic (cf. Anderson, 1991; Feldman, 2000), CP theory has been the major route through which researchers have sought to reformulate our understanding of human rationality (Oaksford & Chater, 2007; Oaksford & Chater, 2009). Could we employ CP theory to model the constructive role of certain judgments? An important difference between CP and QP theories concerns the kind of uncertainty reflected in probability values. In CP theory, probability always reflects uncertainty about the value of a property, which is obviously unknown, but nevertheless objectively exists, independent of our powers of observation (or introspection). In QP theory, there is a kind of uncertainty (arising from pure superposition states), which is not a matter of lack of knowledge, but rather corresponds to a potentiality of which values will be created, after a measurement. Thus, a baseline CP intuition is that a judgment or observation does not have to be constructive, while in QP theory certain judgments are required to be constructive.

We can develop this baseline, CP theory intuition, for the present paradigm. An advert in the present experiments can be interpreted as having a positive or negative content, which can change with time, depending on e.g., extent of processing. Classically, it is assumed that the advert definitely has positive or negative affective content (e.g., activation of positive vs. negative neural pathways), even if it is inaccessible to introspection, prior to a judgment. So, if we are uncertain about the affective content of the advert, then this uncertainty reflects our lack of knowledge of what this objectively existing content is. A measurement (e.g., as prompted by a rating) could just read off the underlying value and so have no constructive role.

We stress that, clearly, there can be judgment models, based on CP theory, which incorporate a constructive role for decisions or judgments. For example, perhaps the process of making a rating forces a transition from weakly dominant probabilities about a stimulus property to strongly dominant ones. Our only point is this: QP theory naturally incorporates a mechanism for the constructive nature of resolving uncertainty, which is highly constrained by the mathematics of QP theory. Can this mechanism be matched to empirical evidence regarding the constructive nature of human judgments? If not, then there are no alternative constructive mechanisms in QP theory we can consider. In CP theory, judgments can easily be made constructive, with assumptions which are fairly straightforward, but, also, more loosely constrained. Therefore, we suggest that, in modeling the constructive role of judgments, the choice of adopting QP theory is well motivated. In other work, we have analogously argued in favor of employing QP theory in cognitive modeling, exactly for those cases for which a formalization based on CP theory works less well (Aerts, 2009; Bruza, Kitto, Nelson, & McEvoy, 2009; Busemeyer & Bruza, 2011; Khrennikov, 2010; Pothos & Busemeyer, 2013; Wang & Busemeyer, 2013).

6. General discussion

At the heart of the present research is the debate on the following issue: are the feelings of subjective awareness we have, relating to choices or preferences or even simple impressions, linked to a constructive process of *creating* some of the relevant information or do they reflect a process of reading off internally generated and pre-existing information? That is, by the time I try to decide whether to have an orange vs. a chocolate bar, or whether I like the painting in front of me, is a corresponding choice or impression value already generated? Or is the process of deciding/articulating an impression constructive, in the sense that it partly generates the relevant cognitive state? Regardless of prior intuitions, there is clearly a challenge in providing robust relevant evidence and a corresponding, rigorous modeling approach.

The present research follows in the steps of influential work arguing for a constructive role of judgment or choice (e.g., Ariely & Norton, 2008; Johnson et al., 2007; Payne et al., 1993; Reyna & Brainerd, 1995; Schwarz, 2007; Sharot et al., 2010). Our work develops this research in several ways. We advance the knowledge of what kind of processes can be constructive: not just choices/preferences, but simpler processes of articulating an impression as well. Existing work concerns evidence for a constructive influence primarily for the former. When there is a choice to be made, perhaps it is easier to accept the idea that the choice alters the underlying preference states as well (cf. Festinger, 1957). But why would there be constructive processes in the case of simply articulating an impression for a stimulus? This is less intuitive and so the present demonstration more surprising. It is worth noting that, with anchoring experiments, evidence suggests that an anchor value reliably affects an absolute judgment only if it is relevant to that judgment (e.g., Chapman & Johnson, 1994; Strack & Mussweiler, 1997). We used unrelated stimuli and the evaluation of the first advert is logically independent of the evaluation of the second advert. This point also applies to possible explanations based on proposals for order effects in e.g., assessing evidence, such as Hogarth and Einhorn's (1992) anchoring and adjustment model.

Theories on how judgments can be constructive often incorporate powerful conceptual intuitions regarding why a constructive influence can arise. For example, in Festinger's (1957) account, there is the idea that constructive processes, as a result of decision making, could arise as a way to minimize feelings of regret about an abandoned option (see also Schwarz, 2007). We also briefly considered some well-known theories directly incorporating an assumption of constructive processes (e.g., Johnson et al., 2007; Reyna, 2008; Reyna & Brainerd, 1995). All these accounts have enabled important theoretical developments, but we discussed our reservations regarding their applicability to the present results.

We considered in detail Hogarth and Einhorn's (1992) theory, which can make predictions on order effects relating to, e.g., a sequence of pieces of evidence, all bearing on a hypothesis. The result of the present experiments concerns the impact of an intermediate judgment on a later judgment, when the order of the corresponding stimuli is the same; thus, the result cannot be explained as an order effect. But, Hogarth and Einhorn's (1992) model can, in principle, accommodate the impact of an intermediate judgment as well, because it distinguishes between evaluations which are EoS vs. SbS. But, the anchoring and adjustment model, at least relative to the model's original parameterization, makes a prediction of no difference for the simple empirical situation we considered, regardless of the inclusion of the intermediate judgment or not. This prediction is inconsistent with the main empirical result of the present study.

A novel contribution of the present work is that we were able to provide a proposal for constructive influences, based on the formal probability framework of QP theory. This perspective complements existing theory in important ways: while in existing theory a constructive influence is an assumption, which can be retained or abandoned with varying degrees of ease (depending on the particular theory), in QP theory a constructive influence for certain kinds of judgments is a fundamental, immutable aspect of the theory. Thus, the application of QP theory to the empirical situation we presented enabled us to lay out the exact sequence of thoughts, as well as describe the new knowledge structures that are created.

The present work also allowed us to further consider the role of formal probability theory in modern cognitive theory. QP and CP theory approaches to cognitive modeling share many characteristics, notably an assumption that cognition can be understood in terms of the principles of a formal probabilistic framework. Thus, a consideration of one approach in a particular theoretical context naturally prompts the question of whether the other might offer an equivalent, alternative explanation. In this case, we noted that CP theory is a theory of epistemic uncertainty, that is, of uncertainty reflecting a lack of knowledge regarding possible outcomes. So, it is arguably less suited to accommodating constructive processes, though this is not to say that CP models cannot be augmented with suitable components. Future work will address whether a CP approach can provide a convincing account of constructive judgments.

One major innovation relating to the use of QP theory in cognitive modeling is that, as noted, QP theory requires a constructive process, whenever there is a transition from a superposition state to a definite one. Thus, QP theory is a formal probability theory (and so retains some of the important advantages for using CP theory in cognitive modeling), but in which constructive cognitive processes can be naturally modeled. An advantage of employing the QP framework is that it enabled us to predict constructive processes in simpler situations of just articulating an impression (an affective evaluation) for a stimulus.

The QP approach we outlined, with minimal assumptions, could be used to make predictions about an interesting interaction, depending on the presence or not of an intermediate rating. These predictions were confirmed across three experiments, and a variety of control conditions. In otherwise identical tasks, with an intermediate rating, the same negative image was rated more negatively, than without the intermediate rating in the PN condition, and analogously for the NP condition. Our results indicate that the intermediate rating did 'something' to the cognitive state and so support the idea that the measurement of internal states can have a constructive role, in the way predicted by the QP approach.

We can elaborate on the psychological insight from the QP approach, by analogy with previous work (e.g., Busemeyer et al., 2011). The cognitive state vector initially coincides with the first advert. Making the intermediate judgment collapses the state either on the purely negative affect ray or the purely positive affect ray. This is like an abstraction process, whereby some of the information about the first advert is forgotten and other information (relating to its affective properties) emphasized. Now, exactly because the two adverts were always chosen to represent opposite affect, being in a pure affective state basically amplified the contrasting impression the second advert made. Thus, we obtained the result that, with the intermediate rating in the PN condition, the second advert was judged more negative and in the NP condition more positive.

Several interesting possibilities for extensions present themselves. First, Hogarth and Einhorn's (1992) model could, in principle, be re-expressed, with QP theory. Hogarth and Einhorn formulated their model in terms of sequences of assessing evidence, either in a step by step manner or in one, final step. These cognitive mechanisms could be translated into QP theory terms, e.g., either through the rotation of a state vector without measurement (end of sequence) or a sequence of measurement projections (step by step). This idea is appealing, but there are technical difficulties. For example, in considering the impact of a sequence of pieces of evidence, some of which may not provide unequivocal impressions, we would need to formalize in exact terms the degree of rotation of the cognitive state vector, depending on evidence strength.

In this work, the specification of the QP theory model was more or less automatic. All representational assumptions, relating to the relative positioning of the rays, follow from the simple design we employed. As the images in the experiment were specified so that their affective content was as clear-cut as possible, so it is the case that the relative positioning of the rays is straightforward. Likewise, it was not necessary to specify in detail the rotation matrices, since all that was needed was an assumption that e.g., a positive advert results in a 'large' rotation of the state vector toward the positive ray. But, more specific formalizations would be necessary for experimental situations involving elements of greater ambiguity. Some work along these lines has already been done (Trueblood & Busemeyer, 2011; Wang & Busemeyer, 2013), though not specifically in the context of constructive judgments. Converging together these separate modeling approaches would be an ambitious objective for further work.

Another important direction for future work concerns understanding in more detail the cognitive mechanisms involved in constructive processes. The formal framework we employed allowed us to specify the quantitative impact of a measurement (articulating an impression) on the cognitive state. What is less clear is whether there may be concomitant changes in, for example, memory or attention. In other words, is a constructive process simply one of changing the relevant cognitive state or does it also generate increased attention or strengthen memory for the cognitive state? These are subtle issues. Some of the theories for constructive judgment are formulated in exactly such (memory or attention) terms, though the implications for memory and attention have generally received less attention in the literature. Note that some possibilities regarding attention/memory can be discounted from the present findings (cf. Anderson, 1981). For example, one suggestion would be that the intermediate rating (somehow) increases attention to the first item. If that were the case, we would expect an increased primacy effect (i.e., the impact of the rating would be to increase the weighting of the first item on the final rating), rather than the observed increased recency effect (i.e., the rating for the second item is more extreme, relative to its affective content; cf. Trueblood & Busemeyer, 2011).

In sum, the idea of superposition (in the QP sense) is novel in psychology. We have argued that the transition from superposition to definiteness is a way to formalize the constructive influence that a process of articulating an impression can be assumed to have. Even though more work is needed regarding both the mathematical and conceptual elaboration of the quantum approach, our results provide a clear empirical case and illustrate a framework for the principled study of such effects.

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Appendix A. Pilot study to select images for Experiment's 1 and 2

A.1. Introduction

The stimuli used in Experiments 1 and 2 were realistic adverts for insurance and smartphones. These product categories were selected, as such products could be advertised using positive, negative and mixed valence images. The purpose of this pilot study was to validate the valence of the selected images and to ensure that they were perceived as unrelated.

A.2. Method

A.2.1. Participants and design

Twelve Swansea University students participated for course credit (8 women, average age 21.75 years).

A.2.2. Stimuli

Two adverts were designed with messages that worked with positive, negative and mixed images. For insurance the message was "Whatever life throws at you… we've got you covered" and for the smartphone the message was "Never miss out… stay in touch". Twenty-four images were selected from various internet sources for having the appropriate valence and because they made sense in the context of the advert and its message. Images were paired so that they would make sense together in a mixed advert.

A.2.3. Procedure

The pilot experiment consisted of three stages. In stage one, participants were told that they would see several images and that for each image, when asked, they should answer the question 'how does this image make you feel?', responding on a nine-point scale, with anchors "1: very unhappy to 9: very happy". Each trial involved the presentation of a single image, followed by a request for rating. Participants were shown the twenty-four images together with twenty-four other images which acted as fillers. Presentation order was randomized for each participant.

In stage two, participants were shown pairs of images and for each pair they were asked to rate the extent to which one image made them think of the second image, responding on a nine-point scale, with anchors "1: not at all to 9: to a very great extent". Each trial involved the presentation of a pair of images, followed by a request for rating. Participants were shown twelve image pairs randomized across participants.

In stage three, participants were shown the same pairs of images as for stage two and for each pair they were asked to rate the extent to which the two images were related, responding on a nine-point scale, with anchors "1: not at all to 9: to a very great extent". Each trial involved the presentation of a pair of images, followed by a request for rating. Participants were shown twelve image pairs randomized across participants.

A.3. Results

The overall mean valence rating for the target adverts was 4.81 ($SD = 1.95$). A paired samples t-test showed that images categorized as negative ($M = 3.09$, $SD = 0.83$) were rated significantly lower than images categorized as positive ($M = 6.53$, $SD = 0.89$; $t(11) = -3.44$, $p < .001$, two tailed; $d = 4.72$).

For advert pairs, the rating for the extent to which one image made them think of the second image was significantly lower than the mid-point of the rating scale (5), using a one sample t-test ($M = 3.05$, $SD = 1.11$; $t(11) = -6.08$,

$p < .001$, two tailed; $d = 1.76$) and the rating for the extent to which the two images were related was also significantly lower than the mid-point of the rating scale (5), using a one sample t-test ($M = 3.31$, $SD = 1.12$; $t(11) = -5.19$, $p < .001$, two tailed; $d = 1.51$).

A.4. Discussion

The results indicate that the selected images were perceived by naïve observers as having the intended positive or negative valence. Furthermore, the relevant image pairs were considered unrelated, as intended.

Appendix B. Examining the predictions from Hogarth and Einhorn's (1992) anchoring and adjustment model for the present empirical paradigm

Adopting Hogarth and Einhorn's notation as much as possible, we want to compare, e.g., S_{PN}^{SbS} with S_{PN}^{EoS}, where SbS = Step by Step process, EoS = End of Sequence process, and PN indicates that we see the positive advert first, followed by the negative advert. Thus, S_{PN}^{SbS} corresponds to the affective rating after seeing the N (negative) advert, given that the P (positive) advert was presented first. In the present context, S_k can be the affective impression of the participants, after considering k adverts ($0 \leqslant S_k \leqslant 1$). Also, $s(x_k)$ is the subjective evaluation of the kth advert. The distinction between S_k and $s(x_k)$ is relevant when considering the impact of different pieces of evidence on a hypothesis, but in the present case we can safely assume that $s(x_k) = S_k$. This means that the positive/negative adverts unequivocally lead to a positive/negative impression, an assumption clearly supported by our relevant pilots (described later). Then, the main equation for Hogarth and Einhorn's model is

$$S_k = S_{k-1} + w_k[S(x_k) - R] \quad (1)$$

This equation tells us that, e.g., participants' affect after considering the kth advert would be the affect just prior to the kth advert, adjusted by the affect from the kth advert, $s(x_k)$. The additional terms in Eq. (1) are R, which is a reference point against which the impact of the kth (in the present case) advert is assessed and w_k, which is an adjustment weight in relation to the kth advert ($0 \leqslant w_k \leqslant 1$. Note that an obvious assumption is that the present task is an *estimation* one, since participants are asked to provide a rating for the adverts, rather than to make a bipolar assessment. This implies that $R = S_{k-1}$ (Hogarth and Einhorn, 1992, p.10). We also assume that S_0 is set by the first advert. This is because participants would clearly have no prior conceptions regarding whether the first advert is positive or negative, prior to seeing the images. Application of the model is then straightforward.

We first apply the model in the SbS case, which, in our experimental paradigm, corresponds to the condition with the intermediate judgment; that is, the first advert is rated by participants, prior to the presentation and rating of the second advert.

$$S_{PN}^{SbS} = S_p + w_N[S(x_N) - S_p] = S_p + w_N[S_N - S_p] \quad (A)$$

In Eq. (A), S_p is the affective impression from the positive advert (presented first) and S_N effectively corresponds to the affective impression of the negative advert, if it were going to be presented in isolation (or first as in the NP order). Thus, the model's prediction, in the PN order, is that the rating for the negative advert will depend on the difference between the affect for the negative and positive adverts ($S_N - S_p$), times an unspecified adjustment weight, and will also depend on the affect of the previously seen positive advert (S_p).

When there is no intermediate rating, we can safely assume that an EoS process is employed, as Hogarth and Einhorn (1992, p.13) state that, when the response mode is EoS (that is, there are no intermediate judgments), the EoS process model is used for short series of cognitively simple evidence items. For an EoS process, Hogarth and Einhorn's model can be written as: $S_k = S(x_1)+w_k[s(x_2,\ldots,x_k) - R]$, where $s(x_2,\ldots,x_k)$ is the combined impact of all the evidence (adverts, in the present case) including the latest piece, following the first piece of evidence. But, in our experiments this simply reduces to the impact of the final advert. Therefore, we have:

$$S_{PN}^{EoS} = s(x_p) + w_N[S(x_N) - S_p] = S_p + w_N[S_N - S_p] \quad (B)$$

It is immediately clear, that, in relation to the present empirical situation, Hogarth and Einhorn's (1992) model predicts that $S_{PN}^{SbS} = S_{PN}^{EoS}$. (As in the above derivation the role of N and P is completely symmetrical, we also have that $S_{NP}^{SbS} = S_{NP}^{EoS}$.) That is, in the case of hardly any evidence (that is, just one advert, presented after an initial advert), Hogarth and Einhorn's (1992) model has to make a baseline prediction of no order difference. By contrast, the key empirical result from the present experiments is that $S_{PN}^{SbS} \neq S_{PN}^{EoS}$ (and likewise that $S_{NP}^{SbS} \neq S_{NP}^{EoS}$). Note that the adjustment weight can depend on the sign of $[S(x_k) - R]$ (Hogarth and Einhorn, 1992, p.14), but in our experiments this was fixed across the critical conditions (following from the above example, in both cases, the relevant quantity is $[S_N - S_p]$). So there is no room for the adjustment weight to vary between the SbS and EoS processes.

We do not claim that there is no parameterization of Hogarth and Einhorn's (1992) model which can lead to the observed empirical results. Letting all the model's parameters vary freely, we have many more model degrees of freedom than the single degree of freedom of the main empirical result. For example, one parameterization which would allow the Hogarth and Einhorn (1992) model to capture the observed empirical finding is if the adjustment weight, w_N, differs between an SbS and EoS process. Arguably, such approaches are too post hoc (in that they are consistent with any possible pattern of results in the present paradigm) to be compelling. A specification of the parameters in Hogarth and Einhorn's (1992) model, in a way consistent with the supporting theory for the model, cannot accommodate the main empirical result in this work. Note also that Hogarth and Einhorn (1992) state that with an SbS process their model always predicts recency, with an EoS process it almost always predicts primacy (it would certainly predict primacy where there is no complex function of evaluating the available evidence). But, the recency Hogarth and Einhorn refer to is defined as $D = S_{ab} - S_{ba} > 0$, where $s(x_a) < s(x_b)$, that is, in words, when b is the stronger piece of evidence, and b is presented last, it has more of an impact on the final rating/impression (this result assumes at least two pieces of evidence, against a background of initial evidence). Thus, Hogarth and Einhorn's recency result concerns comparing two different (SbS; p.48) orders. By contrast, the empirical result we observe concerns increased recency in an SbS order, compared to the *same order* in an EoS process. This last result is not possible within Hogarth and Einhorn's theory and, as far as we can ascertain, is empirically novel.

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